Ideal hydrodynamics and its extensions

Fluctuations, Polarization and gauge theory

G.Torrieri

Based on 2007.09224,2004.10195,1810.12468,1807.02796 , 1701.08263,1604.05291,1502.05421,1112.4086 (PRD,PRC,EPJA) with D.Montenegro,L.Tinti,M.Shokri,L.Gavassino,T.Dore,T.Burch,...

Outline

The usual way to see hydrodynamics

Problems with the usual way small systems

Symmetries-based EFT Hydro as a "chiral perturbation theory"
Good for extensions, not sure about fluctuations

Functional EFT with fluctuations
Putting in "Wilson loops", work in progress, possible small system explanation

The basics

Lifshitz,Landau and the consensus

Some experimental data warmup (Why the interest in relativistic hydro ?)
(2004) Matter in heavy ion collisions seems to behave as a perfect fluid, characterized by a very rapid thermalization

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

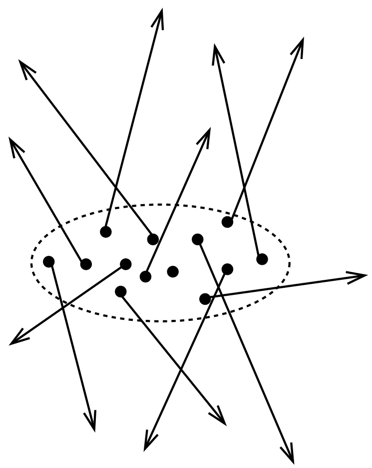
TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

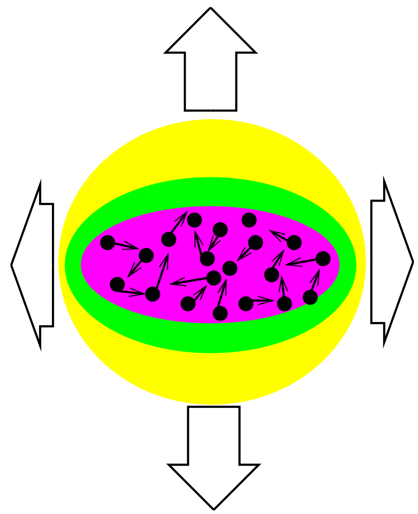
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

The technical details

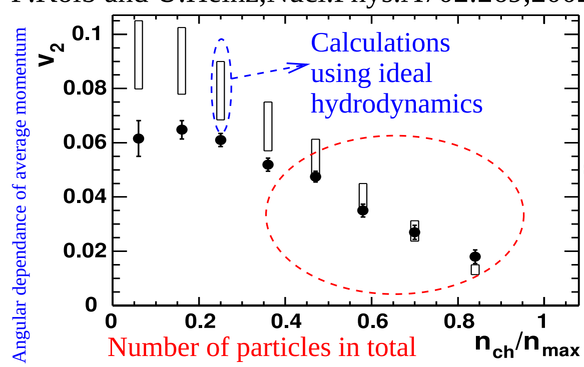
A "dust"
Particles ignore each other, their path is independent of initial shape



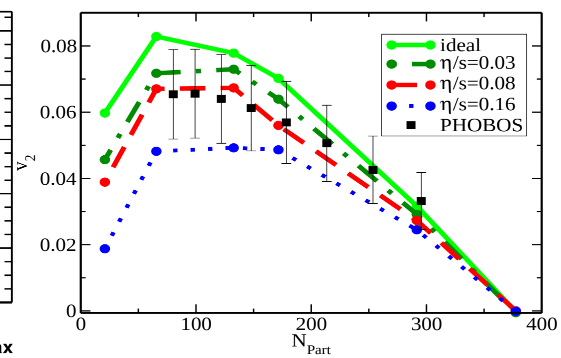


A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)

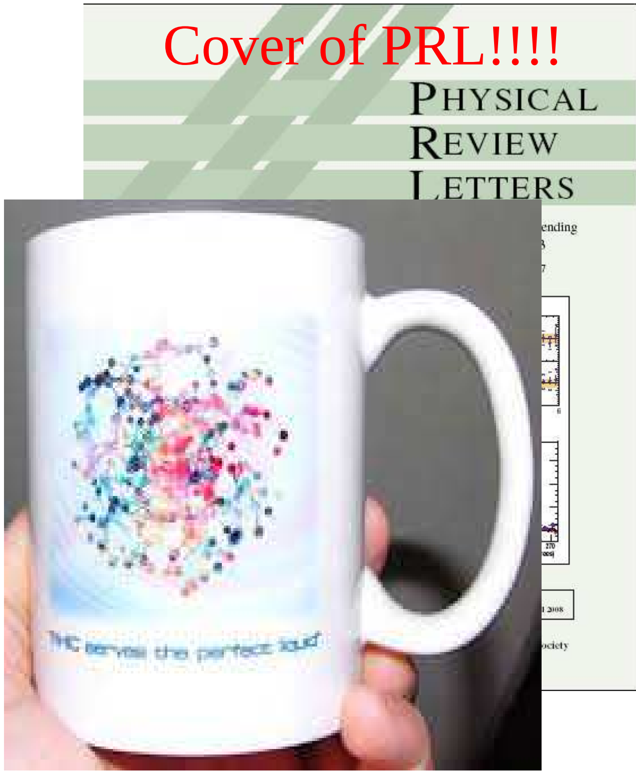
P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007

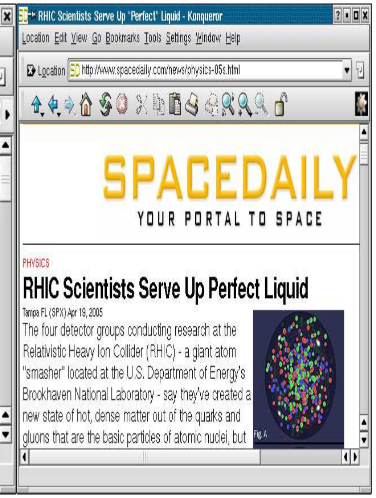


Cover of PRL!!!!



BBC!

SPACE DAILY!



What is (ideal) hydrodynamics (part I)?

Infinite system in equilibrium (relativistic) is characterized by Energy density, Pressure and conserved charge density. Pressure is isotropic (equal in all directions). In this case, Its energy momentum content in the rest frame is characterized by the energy-momentum tensor

$$T^{\mu\nu}_{comoving} = \begin{pmatrix} e(p,\rho) & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

where $e(p,\rho)$ are, in terms of the partition function, the usual relations

$$eV = -\frac{\partial \ln Z}{\partial 1/T} \quad , \quad pV = -T \ln Z \quad , \quad \rho V = -\lambda \frac{\partial \ln Z}{\partial \lambda} \quad \left(\lambda = e^{\mu/T}\right)$$

The energy momentum tensor described in the previous page is only valid in one frame (the rest frame). If this frame, however, is moving with a flow-velocity $u^\mu = \gamma(1, \vec{v})$, then one can use a general Lorentz-transformation

$$\Lambda_\mu^\nu = \begin{pmatrix} \gamma & -v_x\gamma & -v_y\gamma & -v_z\gamma \\ -v_x\gamma & 1+(\gamma-1)\frac{v_x^2}{\vec{v}^2} & (\gamma-1)\frac{v_x v_y}{\vec{v}^2} & (\gamma-1)\frac{v_x v_z}{\vec{v}^2} \\ -v_y\gamma & (\gamma-1)\frac{v_y v_x}{\vec{v}^2} & 1+(\gamma-1)\frac{v_y^2}{\vec{v}^2} & (\gamma-1)\frac{v_y v_z}{\vec{v}^2} \\ -v_z\gamma & (\gamma-1)\frac{v_z v_x}{\vec{v}^2} & (\gamma-1)\frac{v_z v_y}{\vec{v}^2} & 1+(\gamma-1)\frac{v_z^2}{\vec{v}^2} \end{pmatrix}$$

to move to a lab-frame co-moving with u^μ . Then, in the lab frame,

$$T^{\mu\nu} = T^{\alpha\beta}\Big|_{rest} \Lambda_\alpha^\mu \Lambda_\beta^\nu = (e+P)u_\mu u_\nu - p g_{\mu\nu}$$

The conserved charge density becomes a <u>current vector</u> $j^\mu = \rho u^\mu$

Conservation of momentum and Charge always gives us 5 Equations:

$$\underbrace{\partial_\mu T^{\mu\nu} = 0}_{4} \quad , \quad \underbrace{\partial_\mu j^\mu = 0}_{1}$$

However, $T^{\mu\nu}$ has 10 independent components ($4X4$ symmetric matrix), and j^μ has 4. There is generally more to dynamics than conservation laws!

But local equilibrium/isotropy, in some frame, reduces these independent components drastically.

Lets make an approximation: The system is so big w.r.t. the constituents that we can divide it into "infinitesimal volume elements", each of which is infinitely big wrt constituents. Lets furthermore assume that the system expands so slowly wrt the microscopic dynamics that we can disregard microscopic non-equilibrium and just assume that pressure is the only force acting on the system, and the system is always in equilibrium.

In this case, $T^{\mu\nu}$ and j^{μ} are specified by just 6 parameters ($u_{x,y,z}, p, e, \rho$)

$$T^{\mu\nu} = (e+p)u^{\mu}u^{\nu} - pg^{\mu\nu} \quad , \quad j^{\mu} = \rho u^{\mu}$$

Together with the equation of state, we have 6 equations with 6 unknowns. In principle, the system can be solved from any initial conditions

A note on entropy Since

$$s = \frac{dp}{dT} = \frac{p+e-\rho}{T}$$

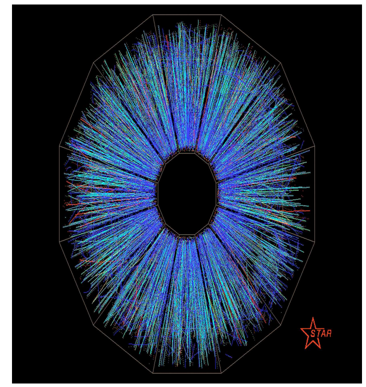
if $e(t), u$ continuus (No shocks or phase transitions!), entropy in an ideal fluid is always conserved, and its possible to rewrite hydrodynamic equations as

$$\underbrace{u^\mu \partial_\mu \left(T u_\nu \right) = 0}_{energy-momentum} \quad , \quad \underbrace{\partial_\mu \left(s u^\mu \right) = 0}_{entropy} \quad , \quad \underbrace{\partial_\mu \left(\rho u^\mu \right) = 0}_{charge}$$

All of hydrodynamics can be rewritten in terms of Speed of sound

$$c_s^2 = -\frac{dP}{de} \quad , \quad s = s(T_0) \exp\left[\int_{T_0}^{T} \frac{dT}{T c_s^2(T)} \right]$$

Why we hope hydrodynamics works to some extent in Heavy ion collisions

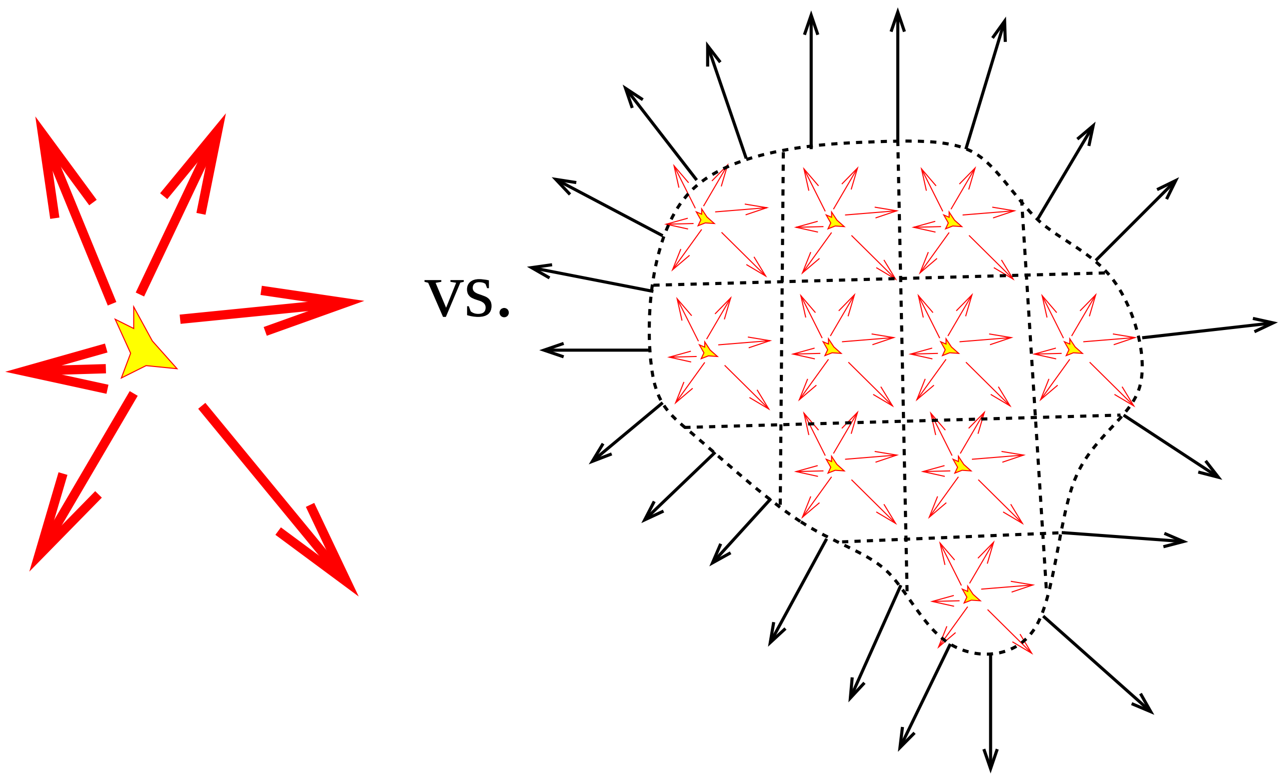
 $\simeq$? 

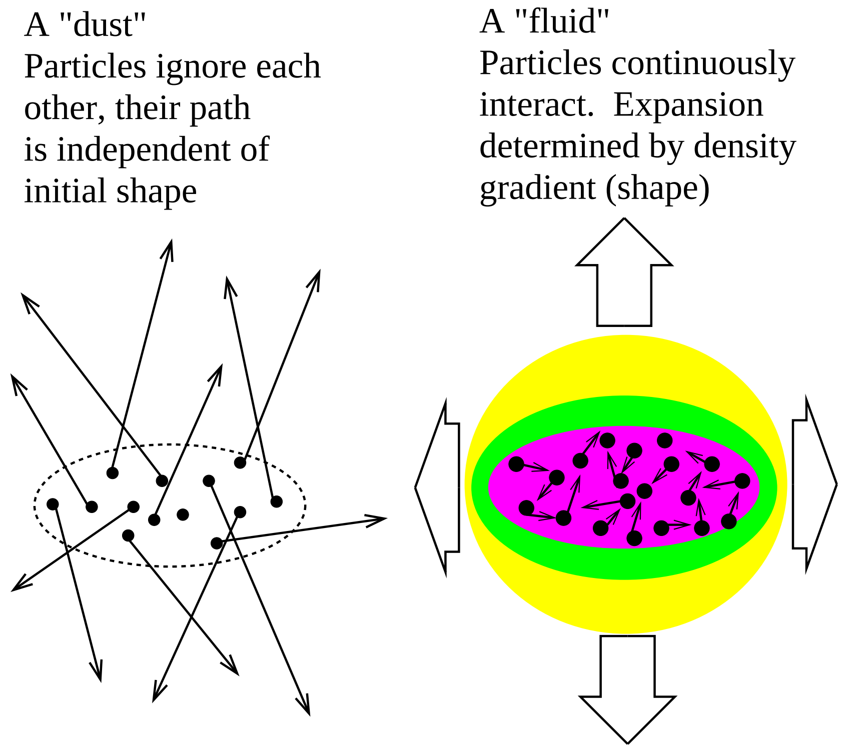
We are in the process of producing and studying the quark gluon plasma, a phase of matter. And of studying the phase transitions and in general the thermodynamics of strongly interacting matter.

But we are creating a very violent and fast explosion of particles. Phase transitions and thermodynamics in general are adiabatic phenomena, changes happen infinitely slowly! The best we can hope for if we want to see QCD thermodynamics is for hydrodynamics to work!

What is not Hydrodynamics:

Equilibration, especially "fake" equilibration, is different from LOCAL equilibration





Signature of local thermalization: Pressure $\rightarrow$ collective flow!
Changes in equation of state, viscosity etc. $\rightarrow$ transition

non-ideal hydro: Deviation from equilibrium "small".
Even if Equilibrium not ideal, we can still find a "flow vector" diagonalizing the symmetric $T_{\mu\nu}$. Eigenvalue will be the Energy density.

$$T_{\mu\nu}u^{\mu} = eu_{\nu}$$

In equilibrium, all other member of $T_{\mu\nu}$will be determined by e, u_{μ}(and the Equation of state). Since we are "approximately" in equilibrium, we can integrate out (Coarse-grain) microscopic degrees of freedom. $T_{\mu\nu}$will then depend on e, u_{μ} and their gradients!

$$T_{\mu\nu} = \underbrace{(p+\rho)u_{\mu}u_{\nu} - pg_{\mu\nu}}_{ideal} + \Pi_{\mu\nu}\left(\partial u, \partial e \partial \rho\right)$$

The form of $\Pi_{\mu\nu}$

- Since we integrated out microscopic dynamics, $\Pi_{\mu\nu} \sim f(\partial u, \partial p, \partial \rho)$ First term in gradient expansion: Only one ∂u(1 term in Taylor)

- These are not independent: $\partial e, \partial \rho$can be put to 0provided we choose a frame at rest with e(Landau Frame) or ρ(Eckart frame). For subsequent discussion we shall do it and forget ρ(Non-ideal Hydrodynamics with ρ never implemented). Hence $u_\mu \Pi^{\mu\nu} = 0$

- 2nd law of Thermodynamics: $\partial_\mu s u^\mu > 0$

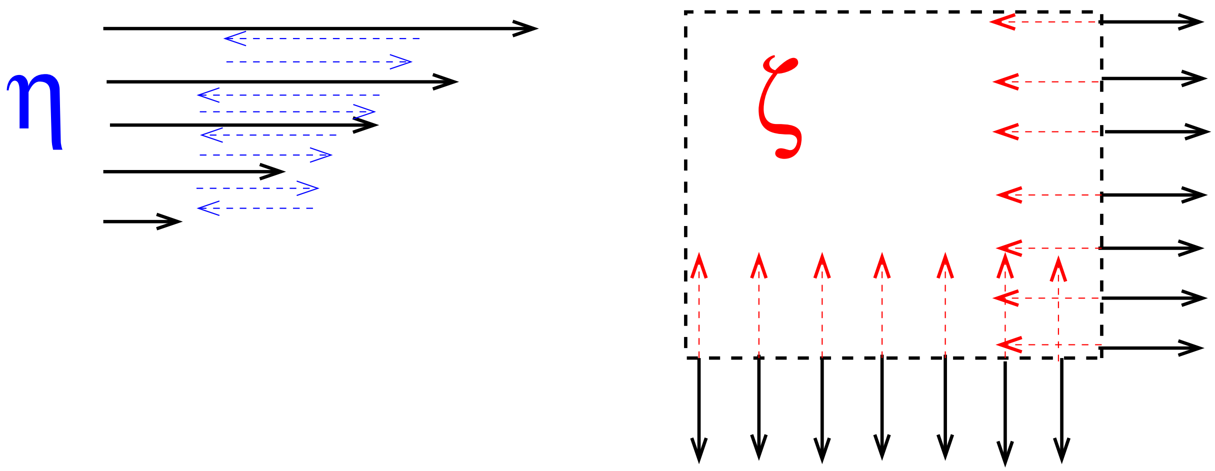
- Lorentz transformations and symmetries: Traceless part ("shear") and Traced part (bulk) have to be independent. Isotropy means that
$$\Pi_{\mu\nu} \sim \underbrace{-}_{Friction} \underbrace{\alpha}_{Equilibrium} \underbrace{\sum \partial u}_{Traceless,traced}$$

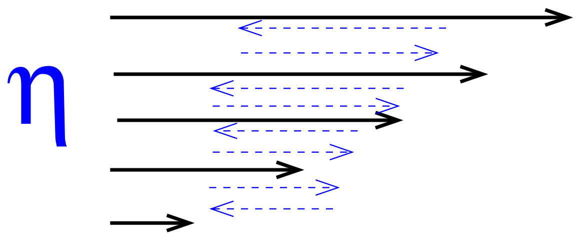
Putting all tese together, we find that the only allowed combination is

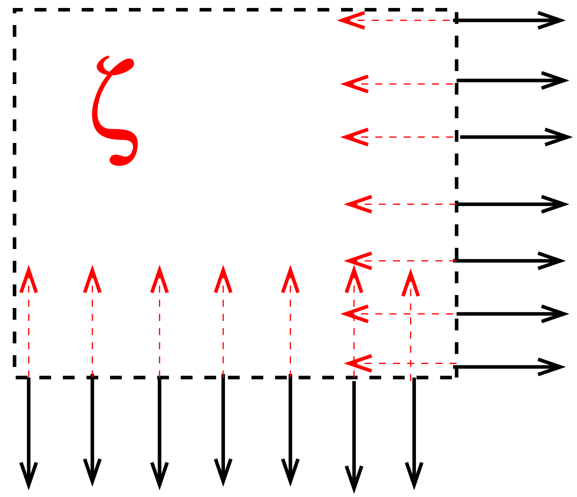
$$\Pi_{\mu\nu} = -\left(\zeta - \frac{2}{3}\eta\right)\partial_\alpha u^\alpha \left(u_\mu u_\nu - g_{\mu\nu}\right)$$

$$-\eta\left(\partial_\mu u_\nu + \partial_\nu u_\mu + u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu\right)$$

where Shear viscosity η and bulk viscosity ζ are new equilibrium parameters!
($6 \to 8$ Equations with $6 \to 8$ unknowns. Complicated but still solvable!)







- Frictions, transforming Gradients into heat
  And hence increase entropy

- Shear viscosity <u>diffusion of momentum</u>, bulk viscosity diffusion across $T_{\mu}^{\mu} = e - 3p$(ie EoS)

- For a conformal gas, ζ (not η)=0

What about conserved currents?

Ideal limit $J^\mu \propto s u^\mu$ otherwise entropy of mixing

Diffusion $J^\mu - s^\mu = q^\mu$ but lose uniqueness of frame

$u^\mu \sim$ **entropy flow** Landau frame, particle diffusion
$u^\mu \sim$ **Current flow** Eckart frame, heat diffusion?

But which is physical? Transport coefficients vary! Isnt flow physical?

Sound waves Expanding Navier-Stokes equations around Static background

$$T_{\mu\nu} = Diag[e,p,p,p] + \delta T_{\mu\nu}(\delta p, \delta e, \delta u_L, \delta u_T)$$

yields dispersion relation for sound waves

$$\partial_t \delta e + ik\delta u_L = J^0$$

$$\partial_t \delta u_L + ic_s^2 k\epsilon + \frac{4}{3T}\frac{\eta}{e_0+p_0}k^2\delta u_L = J^L$$

$$\partial_t \delta \vec{u}_T + \frac{4}{3T}\frac{\eta}{e_0+p_0}k^2\delta \vec{u}_T = \vec{J}^T$$

Sound waves propagate at speed of sound $c_s^2 = dP/de$, diffuse with a power of k^2 and a lenght scale $\sim \eta/(e+p)$. Since Grand-Canonical energies, pressures <u>uncorrelated</u>, linearized relations can be used to extract viscosities from Energy momentum correlations with Quantum-Field theory techniques Kubo formulae

$$\eta = \lim_{w\to 0} \frac{1}{2w} \mathrm{Im} \int dt dx e^{iwt} \left\langle \hat{T}_{xy}(x) \hat{T}_{xy}(0) \right\rangle \quad ,$$

$$\zeta = \lim_{w\to 0} \frac{1}{2w} \mathrm{Im} \int dt dx e^{iwt} \left\langle \hat{T}_{\mu\nu}(x) \hat{T}^{\mu\nu}(0) \right\rangle$$

Usually Kinetic calculations (see next) <u>simpler</u>,through Kubo used in AdS/CFT.

...And we have a problem!

Fourier-Transforming

$$\partial_t \delta \vec{u}_T + \frac{4}{3T}\frac{\eta}{e_0+p_0}k^2 \delta \vec{u}_T = \vec{J}^T$$

we get the dispersion relation

$$w = \frac{4\eta}{3T(e+p)}k^2$$

makes it clear that diffusion speed $w/k \sim k$ grows to ∞ as $k \to \infty$ (wavelength $\to 0$). Our theory has short-wavelength sound waves travelling faster than light. (A common problem to all diffusion-type equations)

Of course this effective long gradient theory should fail for short gradients, but is there a way to see it in effective theory language?

Yes! "2nd order" in Gradient fixes the problem

$$\tau_\pi \partial_t^2 \delta \vec{u_T} + \partial_t \delta \vec{u}_T + \frac{4}{3T}\frac{\eta}{e_0 + p_0} k^2 \delta \vec{u}_T = \vec{J}^T$$

It is intuitively clear that adding a ∂_t^2 (2nd order) term introduces a <u>limiting speed</u> into the dispersion relation that <u>can be made</u> to be $< c$, since then $w^2 + w \sim k^2 + k$ and $w/k \sim k^0$

<u>physically</u>... a <u>timescale</u> for $\Pi_{\mu\nu}$ to <u>turn on</u> given a flow gradient

$$-\tau_\Pi u_\nu \partial_\mu \Pi^{\mu\nu} = \underbrace{\Pi_{\mu\nu}|_{NS}}_{\partial u} + \beta \Pi^{\mu\nu}\Pi_{\mu\nu} + ...$$

But not really second order,through $\tau_\Pi \rightarrow 0 \Leftrightarrow$ Navier-Stokes , but also non-equilibrium thermodynamics

$$
\begin{aligned}
\tau_\Pi \dot{\Pi} + \Pi &= \Pi_{\rm NS} + \tau_{\Pi q}\, q\cdot\dot{u} - \ell_{\Pi q}\,\partial\cdot q - \zeta\,\hat{\delta}_{0,1}\,\Pi\,\theta \\
&\quad + \lambda_{\Pi q}\, q\cdot\nabla\alpha + \lambda_{\Pi\pi}\,\pi^{\mu\nu}\sigma_{\mu\nu} + \hat{\delta}_{0,2}\,\Pi^2 + \hat{\epsilon}_0\, q\cdot q + \hat{\eta}_0\,\pi^{\mu\nu}\pi_{\mu\nu} \\
\tau_q\,\Delta^{\mu\nu}\dot{q}_\nu + q^\mu &= q^\mu_{\rm NS} - \tau_{q\Pi}\,\Pi\,\dot{u}^\mu - \tau_{q\pi}\,\pi^{\mu\nu}\,\dot{u}_\nu \\
&\quad + \ell_{q\Pi}\,\nabla^\mu\Pi - \ell_{q\pi}\,\Delta^{\mu\nu}\,\partial^\lambda\pi_{\nu\lambda} + \tau_q\,\omega^{\mu\nu}\,q_\nu - \frac{\kappa}{\beta}\hat{\delta}_{1,1}\,q^\mu\,\theta \\
&\quad - \lambda_{qq}\,\sigma^{\mu\nu}\,q_\nu + \lambda_{q\Pi}\,\Pi\,\nabla^\mu\alpha + \lambda_{q\pi}\,\pi^{\mu\nu}\,\nabla_\nu\alpha \\
&\quad + \hat{\delta}_{1,2}\,\Pi\,q^\mu + \hat{\eta}_1\,\pi^{\mu\nu}\,q_\nu \\
\tau_\pi\,\dot{\pi}^{<\mu\nu>} + \pi^{\mu\nu} &= \pi^{\mu\nu}_{\rm NS} + 2\,\tau_{\pi q}\,q^{<\mu}\dot{u}^{\nu>} \\
&\quad + 2\,\ell_{\pi q}\,\nabla^{<\mu}q^{\nu>} + 2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\omega^{\nu>\lambda} - 2\,\eta\,\hat{\delta}_{2,1}\,\pi^{\mu\nu}\,\theta \\
&\quad - 2\,\tau_\pi\,\pi_\lambda{}^{<\mu}\sigma^{\nu>\lambda} - 2\,\lambda_{\pi q}\,q^{<\mu}\nabla^{\nu>}\alpha + 2\,\lambda_{\pi\Pi}\,\Pi\,\sigma^{\mu\nu} \\
&\quad + \hat{\delta}_{2,2}\,\Pi\,\pi^{\mu\nu} - \hat{\eta}_2\,\pi_\lambda{}^{<\mu}\pi^{\nu>\lambda} - \hat{\epsilon}_2\,q^{<\mu}q^{\nu>}
\end{aligned}
$$

D.Rischke,B.Betz,Henkel,Niemi,Muronga
Romatshke,Choudhuri,Song,Heinz,...

MANY coefficients not studied at all

Understood fully in conformally invariant theories (Romatschke,Son,...) and partially in pQCD (G. Moore)

Involved (+10 simultaneous equations)

$$\zeta, \eta, \kappa, \quad \underbrace{\tau_\Pi, \tau_q, \tau_\pi}_{\text{relaxation times}} \quad , \underbrace{l_{\Pi q}, l_{q\Pi}, l_{q\pi}, l_{\pi q} \ldots}_{\text{coupling lenghts}}$$

Rephrased as gradient expansion more later

So this is the conventional widsom, but many problems remain

How "universal" is it? Is there a "bottom-up" theory? Link to Microstates?

Different formulations Geroch, Hiscock,Lindblom, now Noronha et al, Kovtun (BDNK) Causal Navier stokes at the price of u_μ "free field" (no local isotropy).

Extensions Spin density, gauge symmetries

Recent experimental data makes these questions "much less philosophical"

So hydrodynamics is an EFT in terms of K and correlators

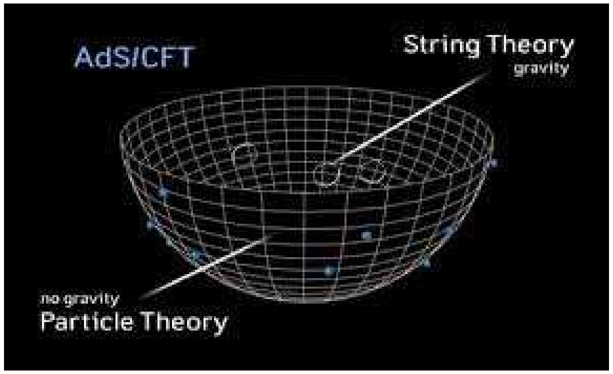
$$\eta = \lim_{k\to 0}\frac{1}{k}\int dx \left\langle \hat{T}_{xy}(x)\hat{T}_{xy}(y)\right\rangle \exp\left[ik(x-y)\right] \quad , \quad \tau_\pi \sim \int e^{ikx}\left\langle TTT\right\rangle, ...$$

This is a <u>classical</u> theory , $\hat{T}_{\mu\nu} \rightarrow \langle T_{\mu\nu}\rangle$ Higher order correlators $\langle T_{\mu\nu}(x)...T_{\mu\nu}\rangle$ play role in transport coefficients, <u>not</u> in EoM (if you know equation and initial conditions, you know the whole evolution!)

As is the case with 99% of physics we know how to calculate rigorously mostly in perturbative limit. But 2nd law of thermodynamics tells us that A Knudsen number of some sort can be defined in <u>any</u> limit as a thermalization timescale can always be defined Strong coupling $\rightarrow$ lots of interaction $\rightarrow$ "fast" thermalization $\rightarrow$ "low" K

e.g. "quantum lower limits" on viscosity? top-down answers
Danielewicz and Gyulassy used the uncertainity principle and Boltzmann equation

$$\eta \sim \frac{1}{5}\langle p\rangle n l_{mfp} \quad , \quad l_{mfp} \sim \langle p\rangle^{-1} \rightarrow \frac{\eta}{s} \sim 10^{-1}$$



KSS and extensions from AdS/CFT (actually any classical Gauge/gravity):
Viscosity$\equiv$ Black hole graviton scattering $\rightarrow \frac{\eta}{s} = \frac{1}{4\pi}$

Von Neuman QM (profound?) or Heisenberg's microscope (early step?)
Both theories not realistic... in a similar way!

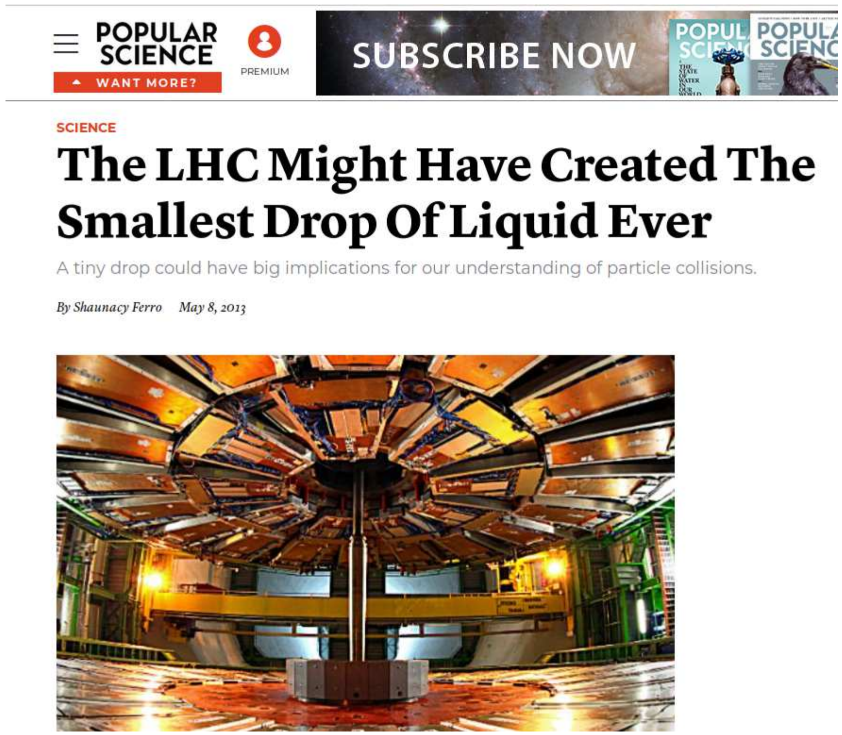
Danielewitz+Gyulassy In strongly coupled system the Boltzmann equation is inappropriate because molecular chaos not guaranteed

KSS UV-completion is conformal,planar, strong

Planar limit and molecular chaos has a surprisingly similar effect: decouple "macro" and "micro" DoFs. "number of microscopic DoFs infinite", "large" w.r.t. the coupling constant!

Too bad experiment threw a spanner in the crucial works of both these pictures, at the same time!

2011-2013 FLuid-like behavior has been observed down to very small sizes, $p - p$ collisions of 50 particles

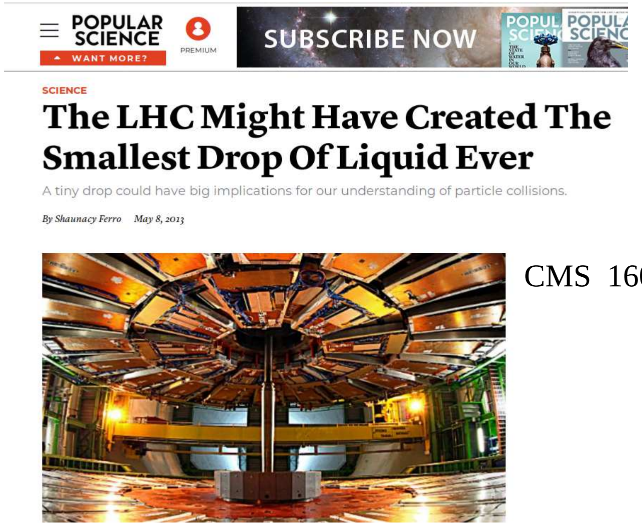


SCIENCE

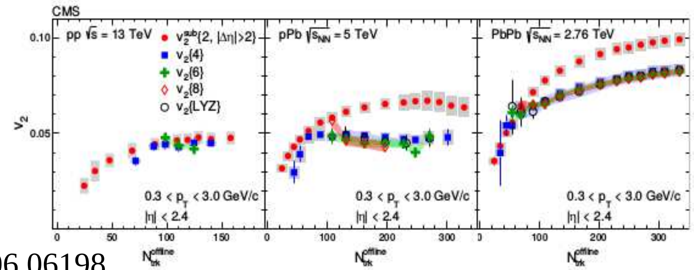
The LHC Might Have Created The Smallest Drop Of Liquid Ever

A tiny drop could have big implications for our understanding of particle collisions.

By Shaunacy Ferro *May 8, 2013*



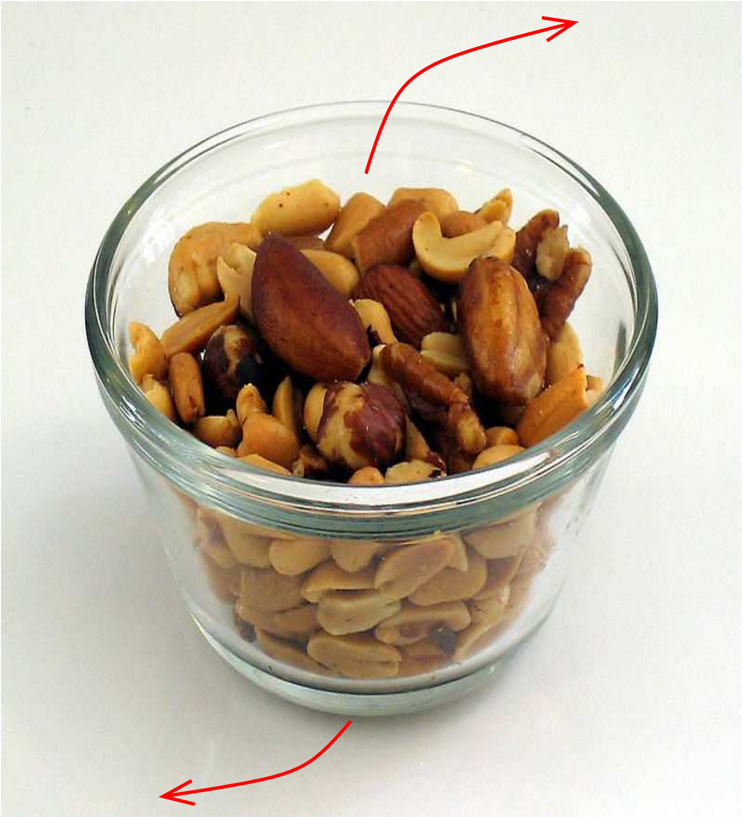
CMS 1606.06198



1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. Thermalization scale $\ll$ color domain wall scale. Also cold atom fluids with 10,000 particles $\sim 1mm^3$

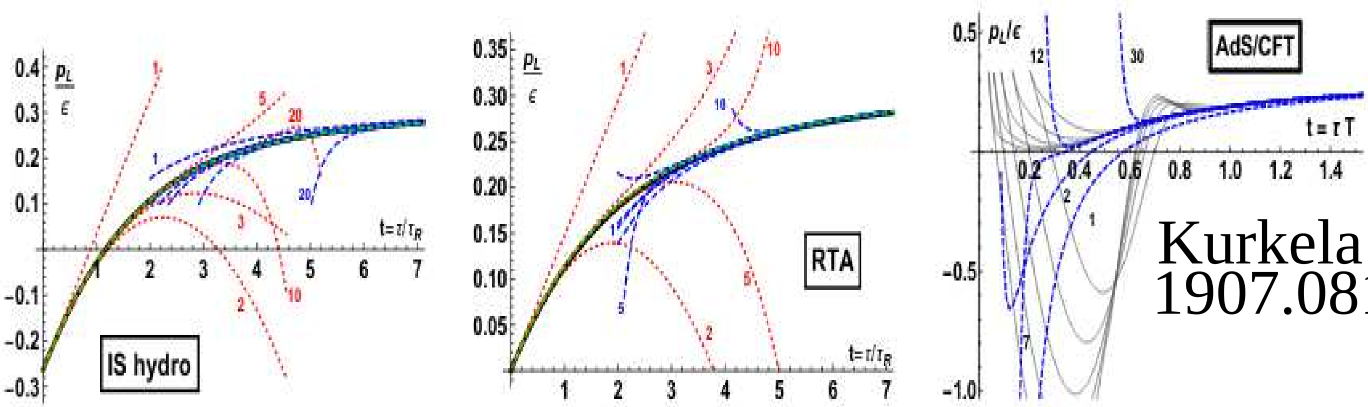
Little understanding of this in "conventional widsom". What si the smallest possible fluid?

The
Brazil
nut effect



Even everyday examples. “How big does a system have to be to exhibit hydrodynamic behavior?” unsolved!

Hydrodynamics in small systems: "hydrodynamization"/"fake equilibrium"
A lot more work in both AdS/CFT and transport theory about "hydrodynamization"/"Hydrodynamic attractors"



Kurkela et al 1907.08101.

Fluid-like systems far from equilibrium (large gradients)! Usually from 1D solution of Boltzmann and AdS/CFT EoMs! "hydrodynamics converges even at large gradients with no thermal equilibrium"

But I have a basic question: ensemble averaging!

- What is hydrodynamics if $N \sim 50$ ...
  - Ensemble averaging , $\langle F(\{x_i\},t)\rangle \neq F(\{\langle x_i\rangle\},t)$ suspect for any non-linear theory. molecular chaos in Boltzmann, Large N_c in AdS/CFT, all assumed . But for $\mathcal{O}(50)$ particles?!?!
  - For water, a cube of length $\eta/(sT)$ has $\mathcal{O}\left(10^9\right)$ molecules,

$$P(N \neq \langle N\rangle) \sim \exp\left[-\langle N\rangle^{-1}(N-\langle N\rangle)^2\right] \ll 1$$

.

- How do microscopic, macroscopic and quantum corrections talk to eac other? EoS is given by $p = T\ln Z$ but $\partial^2 \ln Z/\partial T^2, dP/dV$??

NB: nothing to do with equilibration timescale . Even "things born in equilibrium" locally via Eigenstate thermalization have fluctuations!

And there is more... How does dissipation work in such a "semi-microscopic system"?

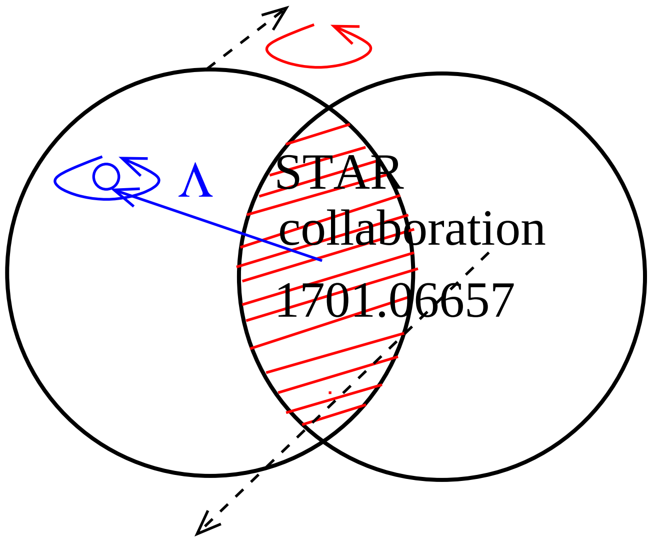
- What does local and global equilibrium mean there?
- If $T_{\mu\nu} \to \hat{T}_{\mu\nu}$ what is $\hat{\Pi}_{\mu\nu}$ Second law fluctuations? Sometimes because of a fluctuation entropy <u>decreases!</u> What is the role of microstates?

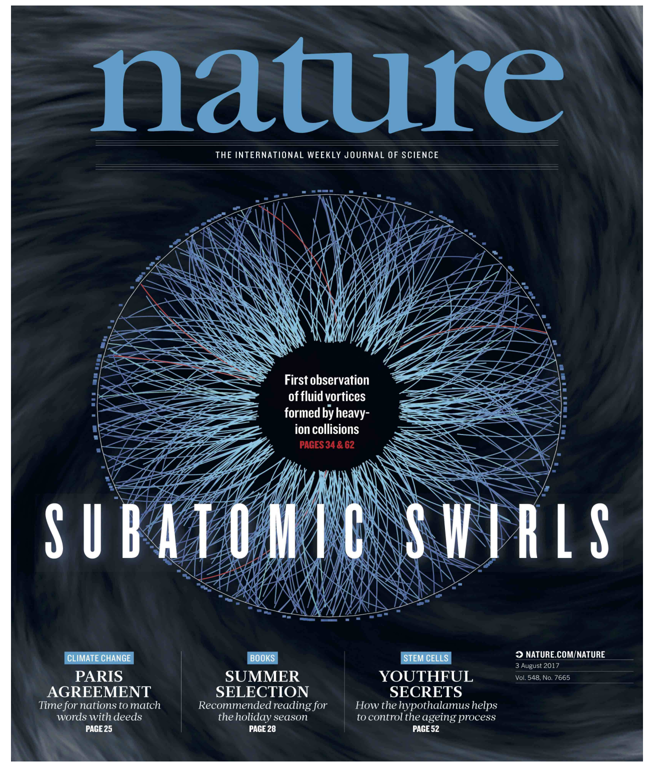
The obvious conclusion is Fluctuations only help dissipation, they are <u>random</u> .
Perhaps $l_{mfp} \geq \mathcal{O}\left(1\right)\left(V/N_{dof}\right)^{1/3}$ or something like this.

Can this be wrong? Can fluctuations <u>help</u> thermalize so smaller systems thermalize <u>faster</u>? if $1/T \sim l_{mfp}$? PERHAPS...

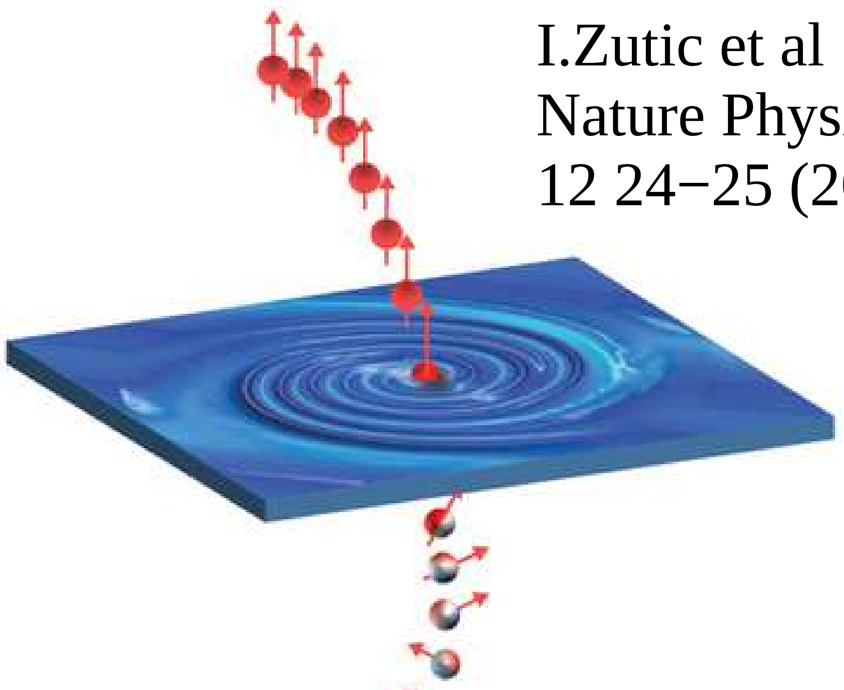
Bottom line: Either hydrodynamics is not the right explanation for these observables (possible! But small/big systems similar!) or we are not understanding something basic about what's behind the hydrodynamics! What do fluctuations do? Just a lower limit to dissipation?

Finally... A spectacular experimental result!

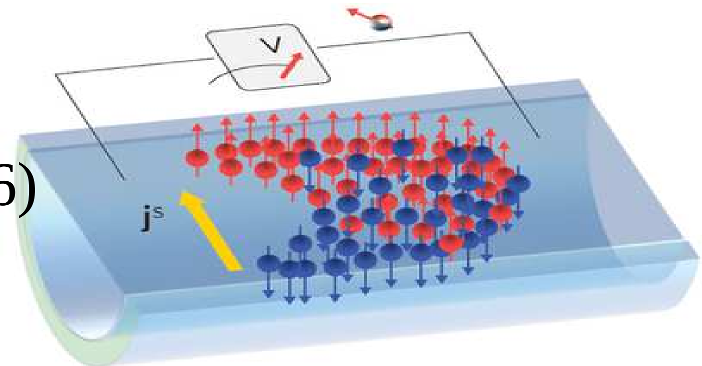


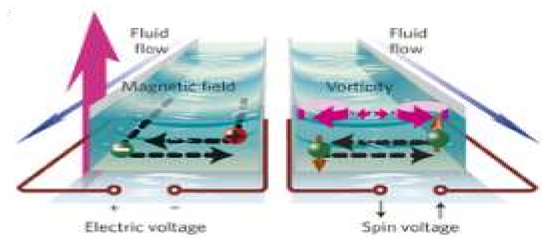


the vorticity associated with the low viscosity fluid observed in AA leaves an imprint on the polarization of the Λ Similar effect seen at low energy!



I.Zutic et al
Nature Physics
12 24−25 (2016)





Ultracold atoms: Zutic, Matos-Abiague, "Spin Hydrodynamics", Nature Physics **12** 24-25 Takahashi et al", Nature Physics **12** 52-56 (2016)

What is ideal hydro? A conceptual difficulty!

Entropy conserved always at maximum at each point in spacetime

Local isotropy in the comoving frame
But polarization non-zero at equilibrium if particles have spin!

Circulation is conserved (Kelvins theorem/Nother current for deformations)
But polarization "absorbs and emits" angular momentum!

Continuum limit when you break up cells, intensive results stay the same
But each particle carries discrete spin unit!

With polarization, only the first has a chance of being realized even in the ideal limit

Raises basic theoretical questions

"vanishing viscosity" but measurable feed-down from collective to microscopic DoFs. Correction to hydro, but not of the usual form

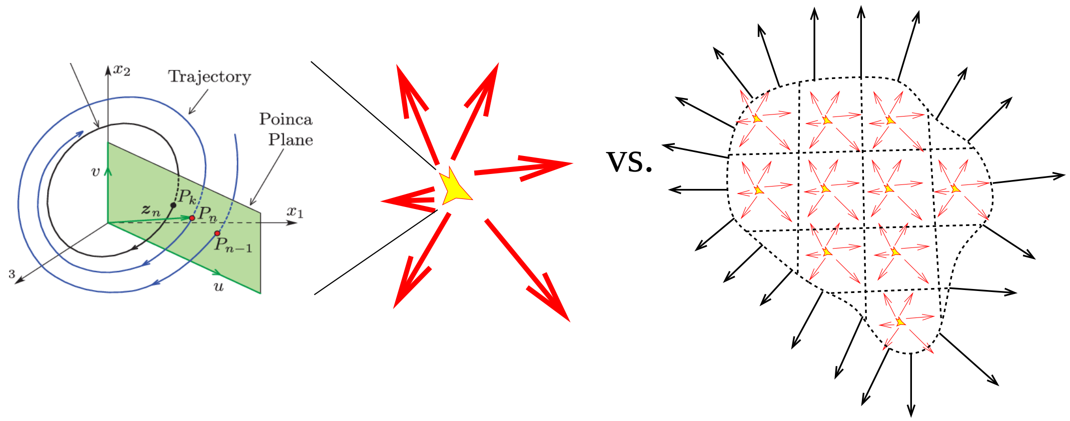
Global/local equilibrium? Relation not clear, system not extensive

Chiral vortical/axial effects not clear weather these are different, complementary,...

The role of pseudo-gauge transformations In many of the models above observables depend on pseudo-gauge choice. does this make sense? basic issue: vorticity "macroscopic", spin "microscopic"

The role of gauge symmetries cannot be avoided if polarization in the early phase. But ambiguities become huge (vorticity/spin ambiguus

The big question



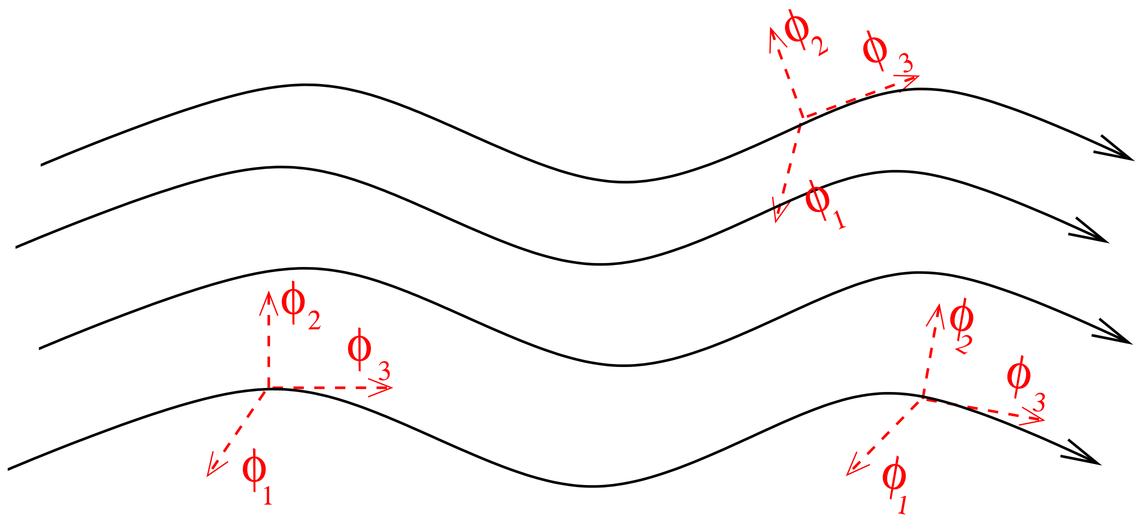
Statistical mechanics says that dynamics is “chaotic enough” that all points in phase space are equally likely

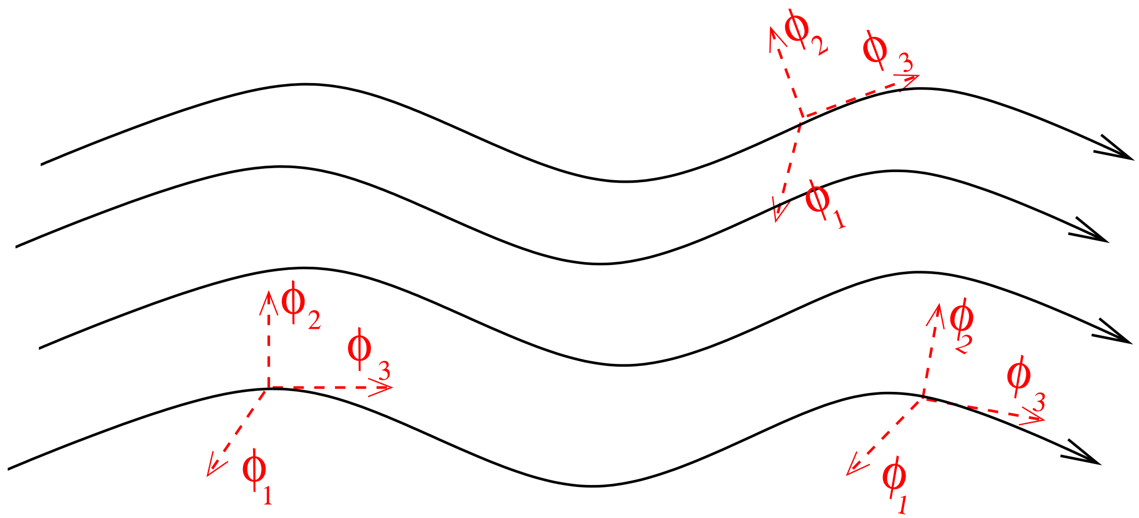
The ideal fluid dynamic limit says that this happens at every point in configuration space of a field

The second should be a limit of the first, and so far it is not!

Hydro as an EFT
Symmetries and extensions

Lets set-up EFT around local equilibrium (Nicolis et al,1011.6396 (JHEP))
Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0, x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)





Local equilibrium$\Rightarrow$ Maximize $s \Rightarrow$ Minimize F $\rightarrow S \underbrace{\Leftrightarrow}_{Legendre} F$

The system is a Fluid if it's Lagrangian obeys some <u>symmetries</u> subject to local minima of free energy. Solutions generally <u>break these</u>, Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons". Causality can be checked by linearizing and finding dispersion relation

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B (actually $b = \sqrt{B}$)
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

(A useful formula is $\frac{db}{d\partial_\mu\phi_I}\partial_\nu\phi_I = u^\mu u^\nu - g^{\mu\nu}$)
Equation of state chosen by specifying $F(b)$. "Ideal": $\Leftrightarrow F(B) \propto b^{2/3}$
b is identified with the entropy and $b\frac{dF(B)}{dB}$ with the microscopic temperature.
u^μ fixed by $u^\mu\partial_\mu\phi^{\forall I} = 0$. Vortices become Noether currents of diffeomorphisms!
This is all really smart, but why?

Hydro as a simplified GR... analogies and differences

4D local Lorentz invariance becomes local SO(3) invariance

Vierbein $g_{\mu\nu} = \eta^{\alpha\beta} e_\mu^\alpha e_\nu^\beta$ is $\frac{\partial x_I^{comoving}}{\partial x_\mu} = \partial_\mu \phi_I$

Killing vector becomes u_μ

$\mathcal{L} \sim \sqrt{-g}\,(\Lambda + R + ...)$ becomes $\mathcal{L} \sim F(B) \equiv f(\sqrt{-g})$

Just cosmological constant, expanding fluid $\equiv$ dS space

Application to acoustic BH on to-do list

Hydrodynamics is based on three scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

l_{micro} stochastic, l_{mfp} dissipative. If $l_{micro} \sim l_{mfp}$ soundwaves

Of amplitude so that momentum $P_{sound} \sim (area)\lambda\,(\delta\rho)\,c_s \gg T$

And wavenumber $k_{sound} \sim P_{sound}$

Survive (ie their amplitude does not decay to $E_{sound} \sim T$) $\tau_{sound} \gg 1/T$

Transport: Beyond Molecular chaos AdS/CFT: Beyond large N_c
It turns out Polarization, gauge symmetries mess this l_{micro} hyerarchy!

Ideal hydrodynamics and the microscopic scale

The most general Lagrangian is

$$L = T_0^4 F\left(\frac{B}{T_0^4}\right) \quad , \quad B = T_0^4 \det B^{IJ} \quad , \quad B^{IJ} = \left|\partial_\mu \phi^I \partial^\mu \phi^J\right|$$

Where $\phi^{I=1,2,3}$ is the comoving coordinate of a volume element of fluid.

NB: $T_0 \sim \Lambda g$ microscopic scale, includes thermal wavelength and $g \sim N_c^2$ (or μ/Λ for dense systems). $T_0 \to \infty \Rightarrow$ classical limit
It is therefore natural to identify T_0 with the microscopic scale!

Kn behaves as a gradient, T_0 as a Planck constant!!!

At $T_0 < \infty$ quantum and thermal fluctuations can produce sound waves and vortices, “weighted” by the usual path integral prescription!

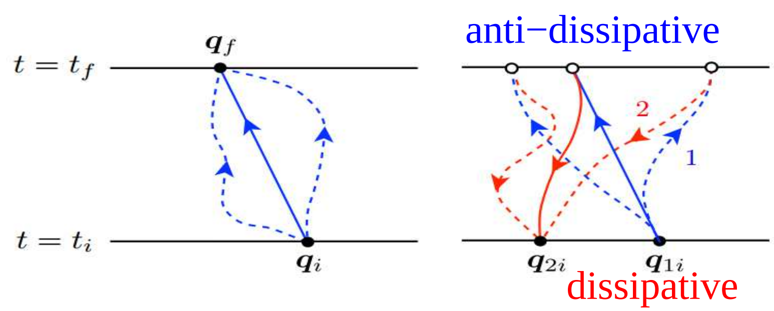
$$L \to \ln \mathcal{Z} \quad \mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B) d^4x \right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial \ldots}$$

$$\left(eg. \quad \left\langle T_{\mu\nu}^{x} T_{\mu\nu}^{x'} \right\rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial g_{\mu\nu}(x) \partial g_{\mu\nu}(x')} \right)$$

$T_0 \sim n^{-1/3}$, unlike Knudsen number, behaves as a ”Planck constant”.EFT expansion and lattice techniques should give all allowed terms and correlators. Coarse-graining will be handled here!

The big problem with Lagrangians... usually only non-dissipative terms
But there are a few ways to fix it. We focus on coordinate doubling (Galley,but before Morse+Feschbach)

Dissipative extension of Hamiltons principle

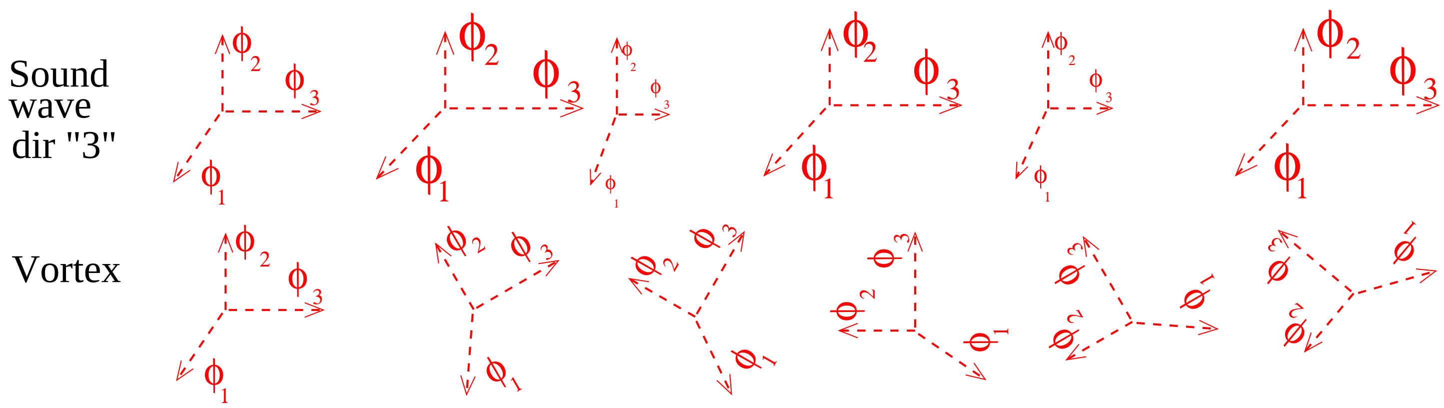


$$L = \frac{1}{2}\left(\underbrace{m\dot{x}^2 - wx^2}_{SHO}\right) \to \underbrace{\left(m\dot{x}_+^{\,2} - wx_+^2\right)}_{\mathcal{L}_1} - \underbrace{\left(m\dot{x}_-^{\,2} - wx_-^2\right)}_{\mathcal{L}_2} + \underbrace{\alpha\left(\dot{x}_+ x_- - \dot{x}_- x_+\right)}_{\mathcal{K}}$$

<u>two</u> sets of equations, one with a damped harmonic oscillator, the other "anti-damped". Navier-Stokes and Israel-Stewart (GT,D.Montenegro, PRD, (2016)) Functional integrals/Lattice also possible!

For analytical calculations fluid can be perturbed around a hydrostatic ($\phi_I = \vec{x}$) background

$$\phi_I = \vec{x} + \underbrace{(\vec{\pi}_L)}_{sound} + \underbrace{(\vec{\pi}_T)}_{vortex}$$



And we discover a fundamental problem: Vortices carry arbitray small energies but stay put! No S-matrix in hydrostatic solution!

$$L_{linear} = \underbrace{\dot{\vec{\pi}}_L^{\,2} - c_s^2(\nabla.\vec{\pi}_L)^2}_{sound \quad wave} + \underbrace{\dot{\pi}_T^{\,2}}_{vortex} + Interactions(\mathcal{O}\left(\pi^3, \partial\pi^3, ...\right))$$

Unlike sound waves , Vortices <u>can not</u> give you "free particles", since they <u>do not propagate</u>: They carry energy and momentum but stay in the same place! Can not expand such a quantum theory in terms of free particles.

Physically: "quantum vortices" can live for an arbitrary long time, and dominate any vacuum solution with their interactions. This does not mean the theory is ill-defined, just that its strongly non-perturbative!
Lattice: Tommy Burch,GT, 1502.05421 In ideal limit, Indications of a 1st order transition between turbulent and hydrostatic phases! Need viscous corrections, fluctuation/dissipation on lattice (BIG project!) But also Polarization might help here!

And chemical potential? Dubovsky et al, 1107.0731

Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

The three length scales of hydro... $\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$

It is clear first 'stochastic' scale controls polarization:

- Vorticity is a "collective" excitation, while polarization is given by microstate counting, $\leftrightarrow$ fluctuations
- Polarization a 2-particle correlation, reducing entropy $f(s_1|s_2)$
- In planar limit fermion polarization typically N_c-suppressed
  Gauge boson polarization not gauge invariant!

Understanding role of polarization is "similar" to understanding role of microscopic correlations/fluctuations in a fluid of vanishing viscosity. Not a conserved quantity so lagrangians help!

Combining polarization with the ideal hydrodynamic limit, defined as

(i) The dynamics within each cell is faster than macroscopic dynamics, and it is expressible only in term of local variables and with no explicit reference to four-velocity u^μ (gradients of flow are however permissible, in fact required to describe local vorticity).

(ii) Dynamics is dictated by local entropy maximization, within each cell, subject to constraints of that cell alone. Macroscopic quantities are assumed to be in local equilibrium inside each macroscopic cell

(iii) Only excitations around a hydrostatic medium are sound waves,vortices

(i-iii) ,with symmetries and EFT define the theory

So how do we implement polarization?

In comoving frame, polarization described by a representation of a "little group" of the volume element.

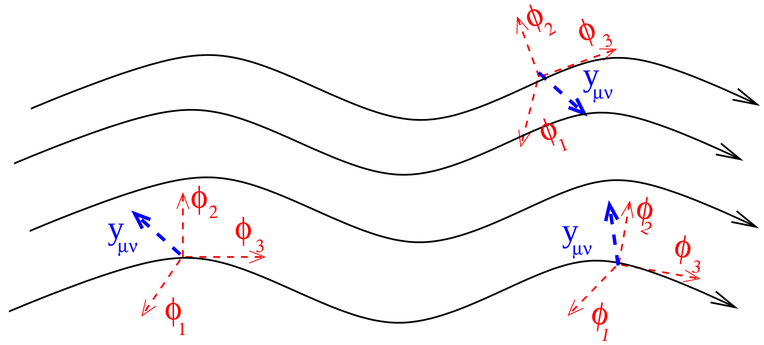
Need local $\sim SO(3)$ charges <u>and</u> unambiguus definition of u^μ $(s^\mu \propto J^\mu)$

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[-\sum_{i=1,2,3} \alpha_i(\phi_I) \hat{T}_i^{\mu\nu}\right]$$

For particle spinor, vector, tensor... repreentations possible.

For "many incoherent particles" RPA means only vector representation remains

Similar to Xin-Li Sheng's "continuus spin"



Chemical shift symmetry, $SO(3)_{\alpha_{1,2,3}} \rightarrow SO(3)_{\alpha_{1,2,3}(\phi^I)}$

$$\alpha_i \rightarrow \alpha_i + \Delta\alpha_i\left(\phi_I\right) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

$y_{\mu\nu} \equiv \mu_i$ for polarization vector components in comoving frame
This way we ensured spin current flows with u^μ. Note that it is not a proper chemical potential (it it would be there would be 3 phases attached to each ϕ_I) as $y_{\mu\nu}$ not invariant under symmetries of ϕ_I. $y_{\mu\nu}$ "auxiliary" polarization field

How to combine polarization with local equilibrium?

Since polarization <u>decreases</u> the entropy by an amount <u>proportional</u> to the DoFs and <u>independent</u> of polarization direction

$$b \to b\left(1 - c y_{\mu\nu} y^{\mu\nu} + \mathcal{O}\left(y^4\right)\right) \quad , \quad F(b) \to F(b,y) = F\left(b\left((1 - c y^2\right)\right)$$

$c > 0$ <u>ferrovortetic</u> $c < 0$ <u>antiferrovortetic</u> (like ferromagnetic but for vorticity!)
Other terms break requirement (i)

First law of thermodynamics,

$$dE = TdS - pdV - Jd\Omega \to dF(b) = db\frac{dF}{db} + dy\frac{dF}{d(yb)}$$

Energy-momentum tensor

Not uniquely defined

Canonical defined as the Noether charge for translations, could be negative because of $\sim \frac{\partial L}{\partial(\partial\psi_i)}\partial\psi_j$

Belinfante-Rosenfeld $\sim \frac{\delta S}{\delta g_{\mu\nu}}$ symmetric independent of spin, no non-relativistic limit

Which is the source for $\partial_\mu T^{\mu\nu} = 0$? Not clear as...

The problem: Too many degrees of freedom

8 degrees of freedom,5 equations $(e, p, u_{x,y,z}, y^{\mu\nu})$. One can include the antisymmetric part of $T_{\mu\nu}$ and match equations but...

No entropy maximization If spin waves and sound waves separated, in comoving volume their ratio is arbitrary... but it should be decided by entropy maximization!

I suspect EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators) will have this problem

Solution clear: make polarization always proportional to vorticity,

$$y^{\mu\nu} \sim \chi(T)(e+p)\left(\partial^{\mu} u^{\nu} - \partial^{\nu} u^{\mu}\right)$$

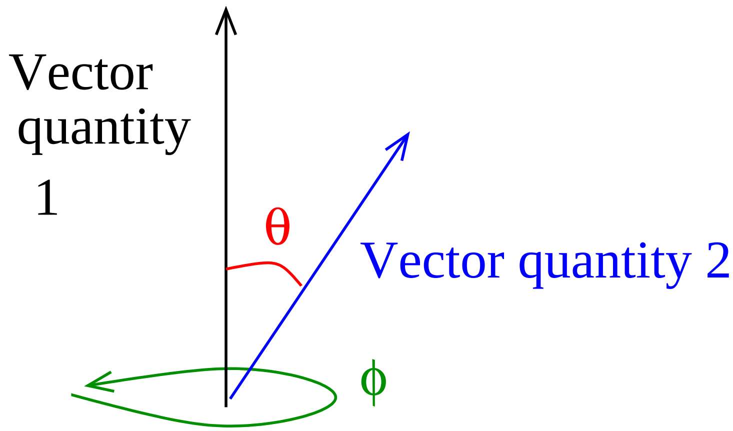
extension of Gibbs-Duhem to angular momentum uniquely fixes χ via entropy maximization. For a free energy $\mathcal{F}$ to be minimized

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V} dV + \frac{\partial \mathcal{F}}{\partial e} de + \frac{\partial \mathcal{F}}{\partial \left[\Omega_{\mu\nu}\right]} d\left[\Omega_{\mu\nu}\right] = 0$$

where $[\Omega_{\mu\nu}]$ is the vorticity in the comoving frame.
THis fixes χ . It also constrains the Lagrangian to be a Legendre transform of the free energy just as in the chemical potential case, in a straightforward generalization of Nicolis,Dubovsky et al. Free energy always at (local) minimum! (requirement (ii))

A qualitative explanation

Instant thermalization means vorticity instantly adjusts to angular momentum, and is parallel to angular momentum. Corrections to this will be of the relaxation type a-la Israel-Stewart



Microscopic physics allows an arbitrary angle between vorticity and polarization. but such systems would have no hydrodynamic limit due to requirement (iii) and the necessity for stability of relaxation dynamics

Radoslaw had a Killing-vector argument, here's a qualitative expanation!

These techniques lead to a well-defined Euler-Lagrange equation of motion

$$\partial_\mu J^\mu_I = 0 \quad , \quad J^\mu_I = 4\,c\,\partial_\nu \left\{ F' \left[\chi\left(\chi + 2\,\partial_{\Omega^2}\chi\right)\omega_{\alpha\beta}\, g^{\alpha\{\mu} P_I^{\nu\}\beta} \right] \right\} -$$

$$-F' \left[u_\rho P_I^{\rho\mu} \left(1 - cy^2 - 2cb\chi\omega^2\,\partial_b\chi \right) \right] - 2c\left(\chi + 2\,\omega^2\,\partial_{\Omega^2}\chi\right) F' \times$$

$$\times \left\{ \left[\chi\,\omega^2 - \frac{1}{b} y_{\rho\sigma}\left(u_\alpha \partial^\alpha K^\rho - u_\alpha \nabla^\rho K^\alpha\right) \right] P_I^{\sigma\mu} - \frac{1}{6b} y_{\rho\sigma} \varepsilon^{\mu\rho\alpha\beta} \epsilon_{IJK} \nabla^\sigma \partial_\alpha \phi^J \partial_\beta \phi^K \right\}.$$

$$P_K^{\mu\nu} = \partial K^\mu / \partial(\partial^\nu \phi^K) \quad , \quad \nabla^\alpha = \Delta^{\alpha\beta}\partial_\beta$$

NB depends on accelleration, so $\Delta S = 0 \Rightarrow \partial_\mu \partial_\nu \frac{\partial F}{\partial(\partial_\mu \partial_\nu \phi^I)} = \partial_\mu \frac{\partial F}{\partial(\partial_\mu \phi^I)}$

J^μ_I : Co-moving total angular momentum components!

Which can be linearized, $\phi_I = X_I + \pi_I$

The "free" (sound wave and vortex kinetic terms) part of the equation will be

$$\mathcal{L} = (-F'(1))\left\{\frac{1}{2}(\dot{\pi})^2 - c_s^2[\partial\pi]^2\right\} +$$

$$+ f\zeta\left\{\ddot{\pi}^i\partial_i\dot{\pi}_j + \ddot{\pi}_i\ddot{\pi}_j + \partial_j\dot{\pi}^i\partial_i\dot{\pi}_j + \partial_j\dot{\pi}_i\ddot{\pi}_j + \right.$$

$$\left. +(2\ddot{\pi}^i\partial_j\dot{\pi}_i - 2\ddot{\pi}_j\partial^i\dot{\pi}_j) + (\ddot{\pi}_i^2 - \ddot{\pi}_j^2) + (\partial_j\dot{\pi}_i^2 - \partial_i\dot{\pi}_j^2)\right\}$$

- Accelleration terms survive linearization

- Vortices and sound wave modes mix at "leading" order. Change in temperature due to sound wave changes polarizability, and that changes vorticity

We decompose perturbation into sound and vortex $\phi_I = \nabla \phi + \nabla \times \vec{\Omega}$

$$\begin{pmatrix} \varphi \\ \vec{\Omega} \end{pmatrix} = \int dw d^3k \begin{pmatrix} \varphi_0 \\ \vec{\Omega}_0 \end{pmatrix} \exp\left[i \left(\vec{k_{\phi,\Omega}} . \vec{x} - w_{\phi,\Omega} t \right) \right]$$

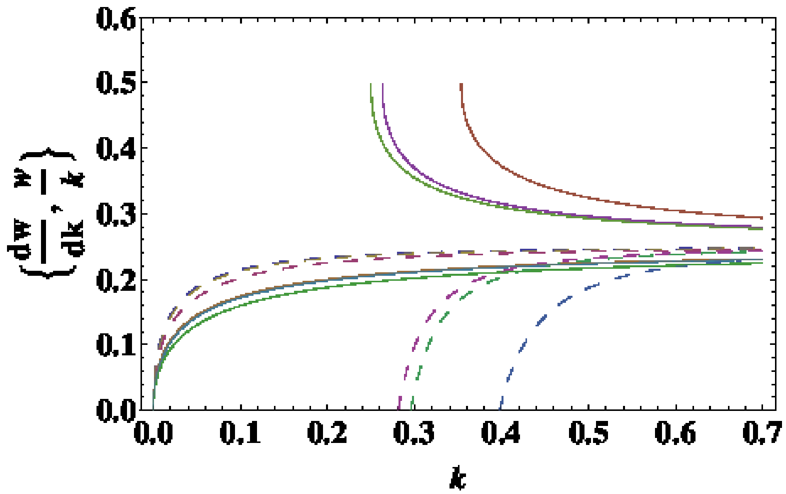
The part parallel to k ("sound-wave") will have a dispersion relation

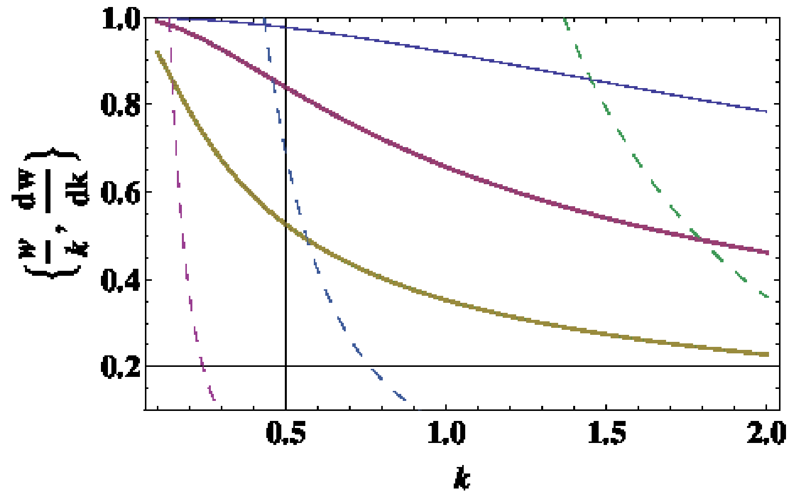
$$w_\phi^2 - c_s^2 k_\phi^2 + 2\beta k_\phi w_\phi^3 = 0$$

The vector part will be

$$(3k_\Omega^2 - 2k_\Omega w_\Omega)_j (\vec{k}_\Omega \times \vec{\Omega}_0)_i w_\Omega^2 + w^4 \Omega = 0$$

Dispersion relations show violation of causality!





Both phase and group velocity will generally go above unity

What I think is going on I: A lower limit of viscosity for polarized hydro

the Free energy $\mathcal{F}$, and hence the local dynamics, is sensitive to an accelleration. As is well-known (Ostrogradski's theorem, Dirac runaway solutions) such Lagrangians are unstable and lead to causality violation. Note that one needs Lagrangians to see this!

To fix this issue, one would need to update the proportionality of y on Ω to an Israel-Stewart type equation

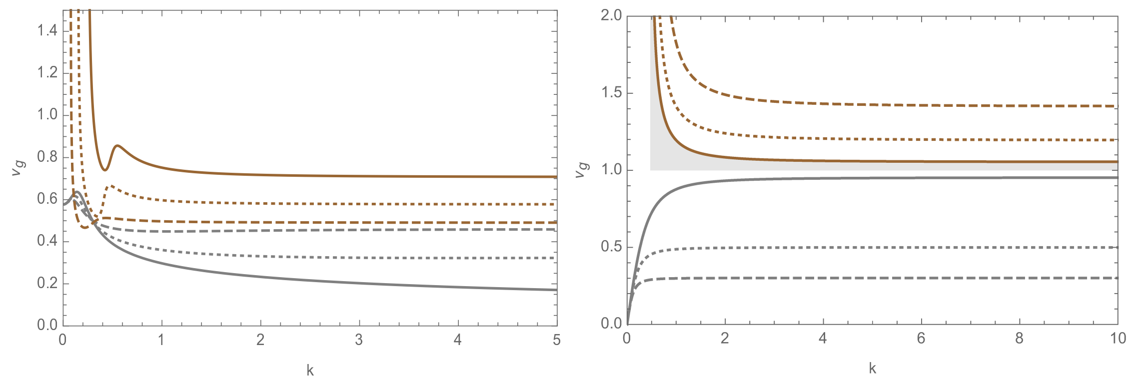
$$\tau_\Omega u_\alpha \partial^\alpha y_{\mu\nu} + y_{\mu\nu} = \chi(T, y)\Omega_{\mu\nu}$$

with an appropriate relaxation time τ_Ω would resolve this issue. Just like with Israel-Stewart, this requires the introduction of new DoFs with relaxation-type dynamics, but, unlike non-polarized hydro,such terms are required from the idea limit

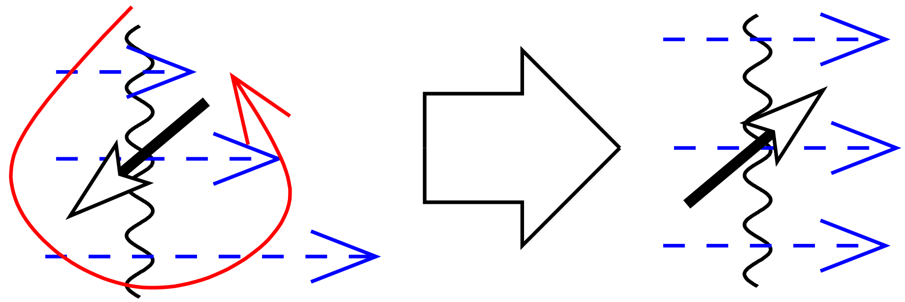
G Torrieri,D Montenegro, 1807.02796 : Polarization are independent DoFs which relax to vorticity

Anti-"Ferrovortetic" fluid non-causal mode ($|dw/dk| \geq 1$) in UV unless

$$\tau_Y^2 \geq \frac{8c\chi^2(b_o,0)}{(1-c_s^2)b_oF'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$



τ_Y regulates quenching of vortices into polarization!

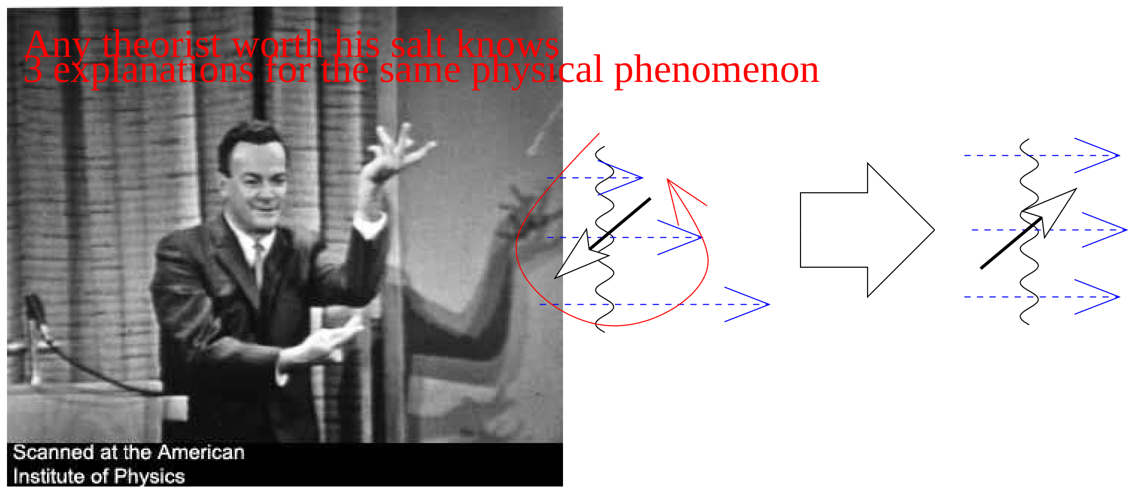


A <u>bottom-up</u> limit on viscosity from polarization!

$$\tau_Y^2 \geq \frac{8c\chi^2(b_o, 0)}{(1-c_s^2)b_o F'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$

<u>Heuristically:</u> At strong coupling vorticity quenches gradients on a timescale $1/T$

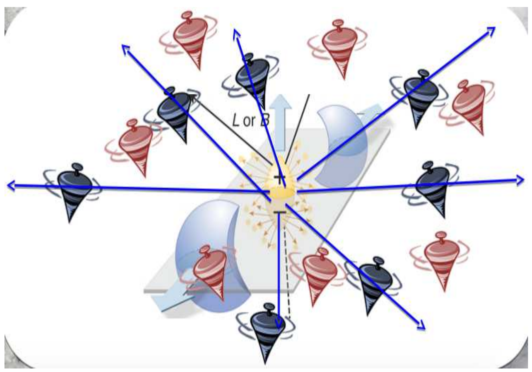
Dispersion relations show violation of causality!

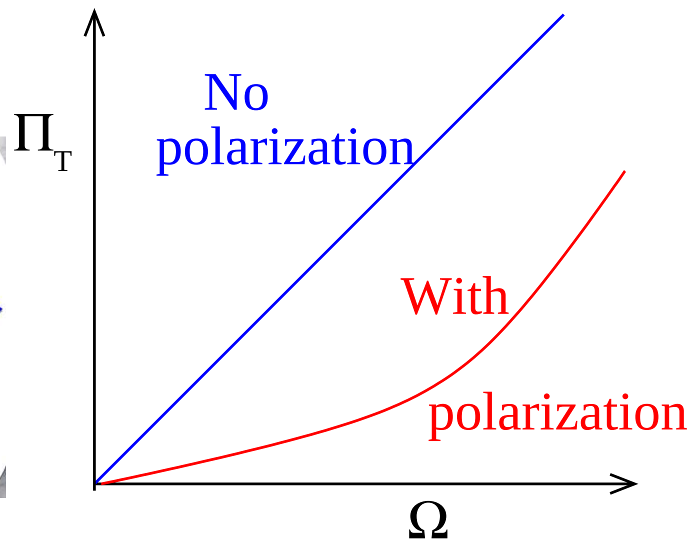


Mathematically : vorticity is an accelleration, instantaneus equilibration to it non-causal. Causality from Relaxation , of spin-flip dynamics "non-local collision term N. Weickgenannt et al , gradient expansion of entropy M.Hongo,S.Shi ,...yield same conclusion! Top-down dissipation constraint from polarizeability!

What I think is going on II

Fluctuation-dissipation: $\tau_\Omega \sim \lim_{\omega\to 0}\omega^{-1}\int dt\,\langle y_{\mu\nu}\Omega_{\mu\nu}\rangle \exp(i\omega t)$





Polarization makes vorticity aquire a "soft gap" wrt angular momentum. At small amplitudes, creating polarization is more advantageus than creating vorticity. This means small amplitude vortices get quenched.

A more rigorous derivation

GT,Montenegro, 2004.10195 A tour de force calculation

τ_Ω, χ related by Kramers-Konig relation

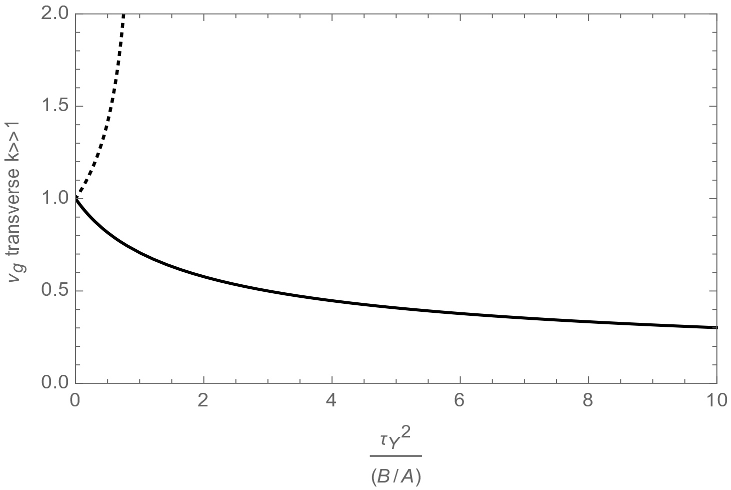
$$\chi = Re[F] \quad , \quad \tau_\Omega = Im[F] \quad , \quad F = Lim_{w \to \infty} \int e^{iwt} \langle [\Omega_{\mu\nu}(t), y_{\mu\nu}(0$$

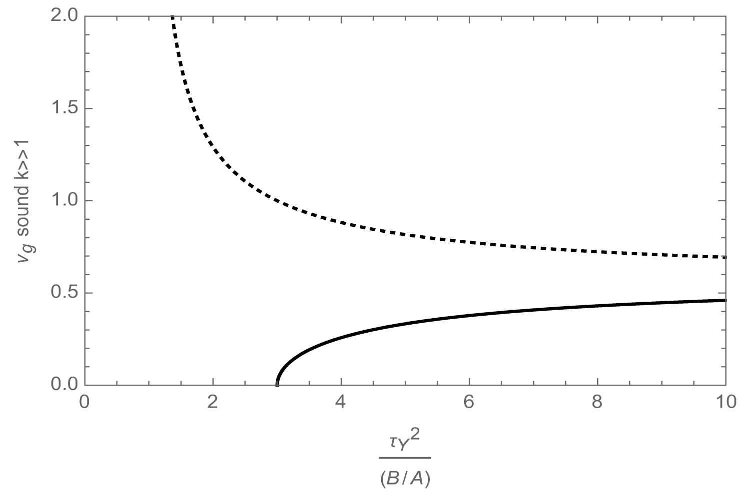
Higgs-like mechanism spontaneus breaking rotation symmetries and giving "mass" to vortices

Tails of corelation functions have same behavior as fluctuating hydrodynamics

Kubo formulae and correlators of both $T_{\mu\nu}, J_I^\mu$

Ferrovortetic fluid   Causal mode in IR remains





But remember that ferromagnetic vaccum is unstable and

$$\langle y_{\mu\nu} \rangle \equiv \lim_{k \to 0} \rho(\omega_{T,L}(k))$$

equivalent to Banks-Casher mode for unstable vacua!

EFT expansion unreliable in this case. Must construct gap equation, fluid with ferro-magnetic like (ferrovortetic) phase transition!

Phenomenology

Dilepton and photon polarization sensitive to spin density in the early phase, if measurable (can experiment double as a SG detector for dileptons? Speranza et al, 1802.02479,Baym et al, 1702.05906

Vector mesons could help in both vector meson and longitudinal spin discrepancies with hydro!

- Longitudinal polarization timescale different from transverse (longitudinal spin might not be in equilibrium)
- Cooper-Frye formula cannot work if spin and vorticity already present. Coalescence Wigner functions? Coherence?

Vector mesons might allow us to test this quantitatively since there is a more information in their decay distribution...

$$W\left(\theta,\phi\right) \sim \cos^2\theta\rho_{00} + \sin^2\theta\left(\frac{\rho_{11}+\rho_{-1-1}}{2}\right) - \sin 2\theta\left(\frac{\cos\phi Re\rho_{10} - \sin\phi Im\rho_{10}}{\sqrt{2}}\right) -$$

$$+\sin 2\theta\left(\frac{\cos\phi Re\rho_{-10} + \sin\phi Im\rho_{-10}}{\sqrt{2}}\right)$$

$$\rho = U_{\theta,\pi}^{-1}\begin{pmatrix} 1+n_8 & \sqrt{3}(n_1 - in_2) & 0 \\ \sqrt{3}(n_1 + in_2) & 1+n_8 & 0 \\ 0 & 0 & 1-2n_8 \end{pmatrix} U_{\theta,\phi}$$

To verify would neet both both θ and ϕ of vector mesons and photons.

$$\Psi_S^M = \sum_{S_1 S_2} \left(C_{S_1 S_2}^L\right)_S \Psi_{S_1}^{q_1}\Psi_{S_2}^{2_2} \quad , \quad L \equiv Vorticity$$

Coherence: $\rho^2 = \rho$ Vorticity: $S_{1,2}$ vs L . Work with Kayman Jhosef

Theory: Global/local equilibrium: 2007.09224

`https://www.youtube.com/watch?v=oLYouz0YMHM` (2 days ago)

Need to include fluctuations and pseudo-gauge dependence. Only way I can see it possible is Zubarev hydrodynamics

expanded around equilibrium $L \sim \ln Z \quad , \quad Z = Z_{T_0} \times Z_{\Pi}$

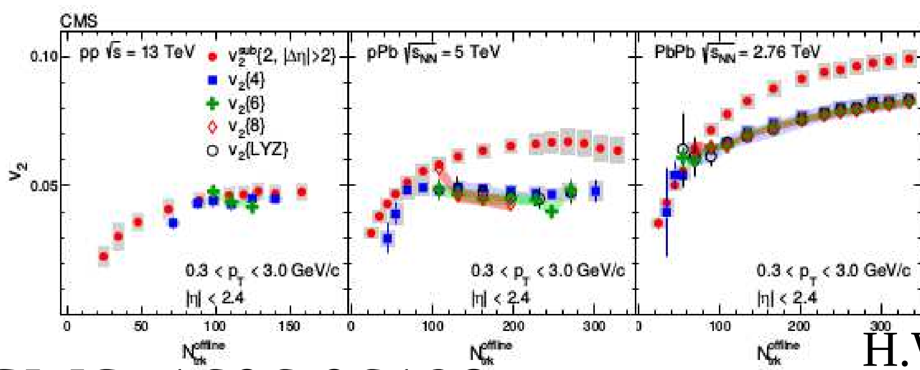
$$\hat{\rho}_{T_0} = Z_{T_0}^{-1} \exp\left[-\beta_\mu d\Sigma_\nu \left(\hat{T}^{\mu\nu} - \hat{n}^\nu - \omega^{\mu\nu\alpha} \hat{J}_\alpha\right)\right]$$

Each part of the free energy is pseudo-Gauge dependent, but free energy invariant! $\hat{\Pi}_{\mu\nu}$ also an operator determined by Crooks theorem

$$Z_\Pi \Rightarrow \frac{P(W)}{P(-W)} \sim \exp\left[\Delta S(\langle \Pi \rangle)\right]$$

Any local equilibrium state will tend to global equilbrium via fluctuations obeying detailed balance. Speaking of pseudo-gauge symmetries...

Another spectacular experimental result



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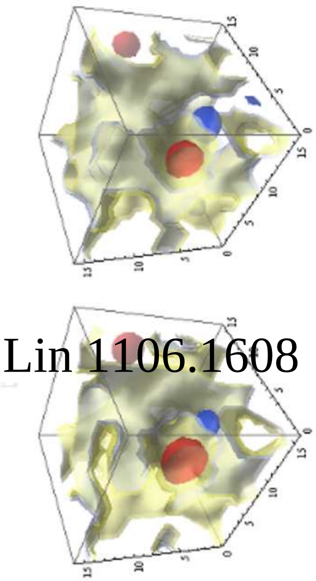
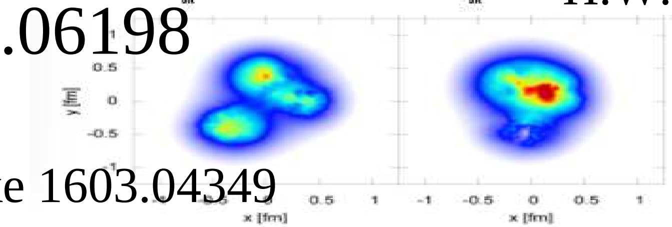


H.W.Lin 1106.1608



BSchenke 1603.04349

1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. So mean free path is really small. What about thermal fluctuations? Nothing here is infinite, not even N_c Also hydro applicability scale below color domain scale. colored hydro?

The formalism we introduced earlier is ok for quark polarization but not gluon polarization: Gauge symmetry means one can exchange locally angular momentum states for spin states. So vorticity vs polarization is ambiguus. Separation in optics, parton spin structure requires a preferred static frame,different from comoving frame

Using the energy-momentum tensor for dynamics is even more problematic for spin $T_{\mu\nu}$ aquires a "pseudo-gauge" transformation

$$T_{\mu\nu} \to T_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda} \right)$$

Φ fully antisymmetric. T.Brauner, 1910.12224:
$\phi \to \phi + \zeta(x), x_\mu \to x_\nu + \omega_\mu(x), S \to S$ But in a gauge theory, pseudo-Gauge transformations are gauge transformations ($\hat{\Pi}_\mu \to \hat{\Pi}_\mu + \hat{A}_\mu$)!
Large gauge configurations change $T^{\mu\nu}$

From global to gauge conserved currents

A reminder: Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

Generalization from $U(1)$ to generic group easy

$$\alpha \to \{\alpha_i\} \quad , \quad \exp(i\alpha) \to \exp\left(i \sum_i \alpha_i \hat{T}_i\right)$$

One subtlety: Currents stay parallel to u_μ but chemical potentials become adjoint, since rotations in current space still conserved

$$y = J^\mu \partial_\mu \alpha_i \to y_{ab} = J_a^\mu \partial_\mu \alpha_b$$

Lagrangian still a function of $dF(b, \{\mu\})/dy_{ab}$, "flavor chemical potentials"

If color was just a global symmetry same thing happens see CFL literature!
But need to covariantize w.r.t. local gauge symmetries

From global to gauge invariance! Lagrangian invariant under

$$\{y_{ab}\} \to y'_{ab} = U^{-1}_{ac}(x) y_{cd} U_{db}(x) \quad , \quad U_{ab}(x) = \exp\left(i \sum_i \alpha_i(x) \hat{T}_i \right)$$

However, gradients of x obviously change y .

$$y_{ab} \to U^{-1}_{ac}(x) y_{cd} U_{bd}(x) = U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} U^{-1}_{fg} \partial_{\mu} \alpha_g U_{bg} =$$

$$= U^{-1}(x)_{ac} J^{\mu}_{f} U_{cf} \partial_{\mu} \left(U^{-1}_{fg} \alpha_d U_{bd}(x) \right) - J^{\mu}_{a} \left(U \partial_{\mu} U \right)_{fb} \alpha_f$$

Only way to make lagrangian gauge invariant is

$$F\left(b, J^{\mu}_{j} \partial_{\mu} \alpha_i\right) \to F\left(b, J^{\mu}_{j} \left(\partial_{\mu} - U(x) \partial_{\mu} U(x) \right) \alpha_i \right)$$

Which is totally unexpected, profound and <u>crazy</u>

The swimming ghost!

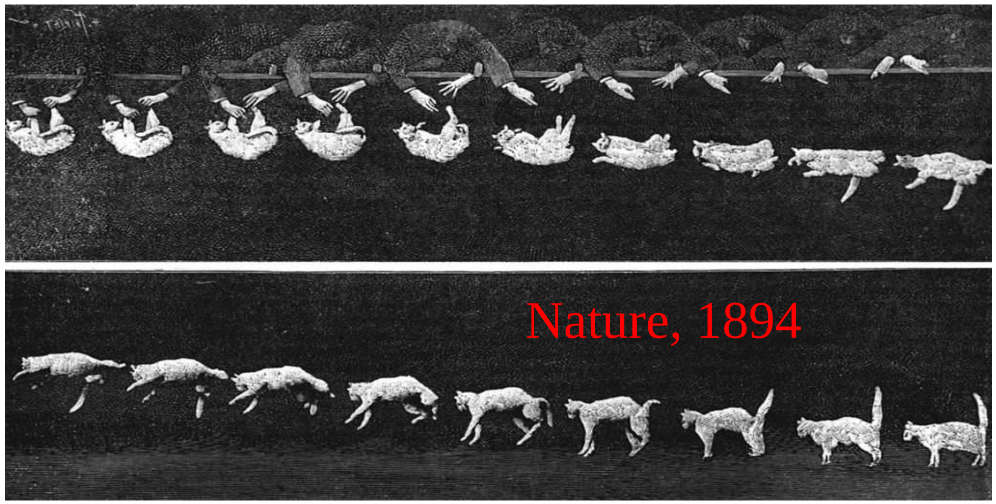
$$F\left(b, J_j^{\mu}\partial_\mu \alpha_i\right) \to F\left(b, J_j^{\mu}\left(\partial_\mu - U(x)\partial_\mu U(x)\right)\alpha_i\right)$$

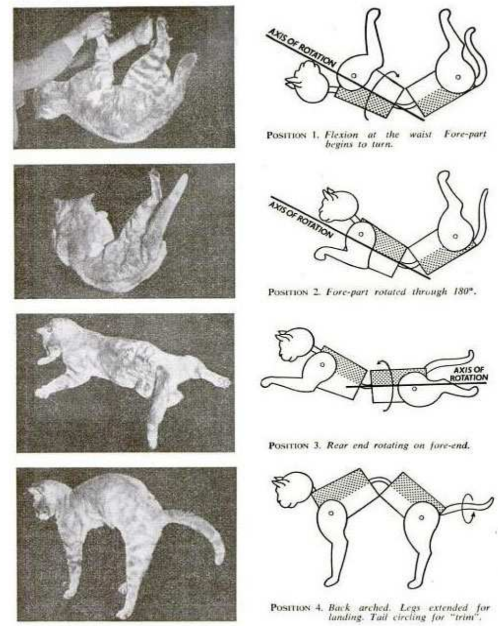
Means the ideal fluid lagrangian depends on velocity!. no real ideal fluid limit possibl
the system "knows it is flowing" at local equilibrium! NB: For U(1)

$$\hat{T}_i \to 1 \quad , \quad y_{ab} \to \mu_Q \quad , \quad u_\mu \partial^\mu \alpha_i \to A_\tau$$

So second term can be gauged to a redefinition of the chemical potential (the electrodynamic potentials effect on the chemical potential).

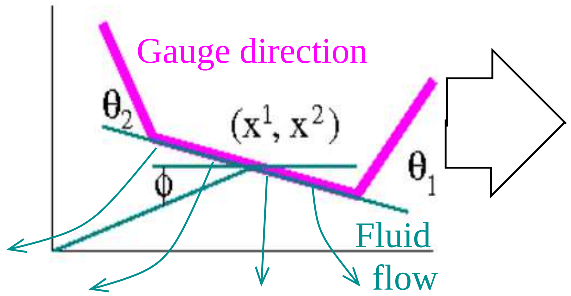
Cannot do it for Non-Abelian gauge theory, "twisting direction" in color space It turns out this has an old analogue...





S. Montgomery (2003): How does a cat always fall on its feet without anything to push themsevles against? The shape of spaces a cat can deform themselves into defines a "set of gauges" a cat can choose without change of angular momentum.

Purcell,Shapere+Wilczek,Avron+Raz : A similar process enables swimmers to move through viscous liquids with no applied force



Now imagine each fluid cell filled with a "swimmer", with arms and legs outstretched in "gauge" directions...

In ideal limit all currents proportional to u_μ . But gauge symmetry requires "ghost" excitations, proportional to gradients of currents, to not be physical. So free energy HAS to depend on flow.

Classic on this, B. Bistrovic, R. Jackiw, H. Li, V. P. Nair and S. Y. Pi, Phys. Rev. D **67**, 025013 (2003) , "NonAbelian fluid dynamics in Lagrangian formulation," missed this subtlety as no local equilibrium defined!

Whats going on? A more statistical mechanics perspective

We perturb the hydrostatic limit, where $\phi_I = X_I$, and isolate a transverse mode (vortex) and a longitudinal mode (sound wave)

$$\phi_I = X_I + \vec{\pi}_I^{sound} + \vec{\pi}_I^{vortex} \quad , \quad \nabla . \vec{\pi}_I^{vortex} = \nabla \times \vec{\pi}_I^{sound} = 0$$

Since the derivative of the free energy w.r.t. b is positive, sound waves and vortices do "work". Let us now assume the system has a "color chemical potential" in some direction Let us change the color chemical potential in space according to

$$\Delta\mu(x) = \sum_i \left(\mu_i(x)^{swim} + \mu_i(x)^{swirl} \right) \hat{T}_i \quad , \quad \nabla_i . \mu_i^{swim} = \nabla_i \times \mu_i^{swirl} = 0$$

Because of gauge redundancy, the derivatives of the free energy with respect to color ("color susceptibility") will typically be negative. So the two can balance!!!!

But this breaks the "hyerarchy" of statistical mechanics

It mixes micro and macro perturbations!

In statistical mechanics, what normally distinguishes "work" from "heat" is coarse-graining, the separation between micro and macro states. Quantitatively, probability of thermal fluctuations is normalized by $1/(c_V T)$ and microscopic correlations due to viscosity are $\sim \eta/(Ts)$. Since for a usual fluid, there is a hyerarchy between microscopic scale, Knudsen number and gradient

$$\frac{1}{c_V T} \ll \frac{\eta}{(Ts)} \ll \partial u_\mu$$

Gauge symmetry breaks it, since it equalizes perturbations at both ends of this!

Is there a Gauge-independent way of seeing this? Perhaps!

One can write the effective Lagrangian in a Gauge-invariant way using Wilson-Loops . But the effective Lagrangian written this way will have an infinite number of terms, in a series weighted by the characteristic Wilson loop size. For a locally equilibrated system, this series does not commute with the gradient. Just like with Polymers, the system should have multiple anisotropic non-local minima which mess up any Knuden number expansion. Some materials are inhomogeneus and anisotropic at equilibrium, YM could be like this!

Lattice would not see it , as there are no gradients there. There is an entropy maximum, and it is the one the lattice sees. The problems arise if you "coarse-grain" this maximum into each microscopic cell and try to do a gradient expansion around this equilibrium, unless you have color neutrality.

Conclusions

Hydrodynamics is not a limit of transport, AdS/CFT or any other microscopic theory

Hydrodynamics is an EFT built around symmetries and entropy maximization and should be treated as such

Once you realize this , generalizing it to theories with extra DoFs, symmetries etc. becomes straight-forward.

Lots of things to do Gauge symmetry looks particularly interesting!

Linking to statistical mechanics

Micro Macro and fluctuations

Landau and Lifshitz (also D.Rishke,B Betz et al): Hydrodynamics has <u>three</u> length scales

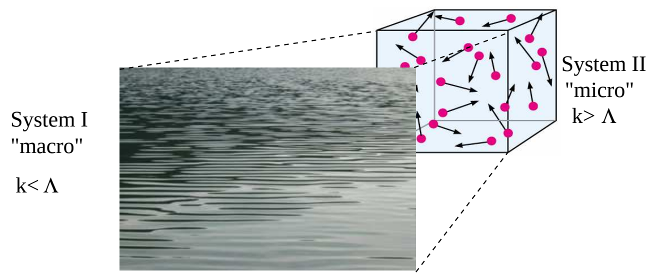
$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

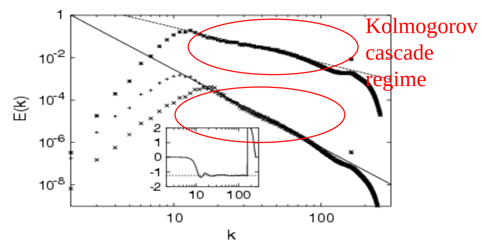
Weakly coupled: Ensemble averaging in Boltzmann equation good up to $\mathcal{O}\left((1/\rho)^{1/3}\partial_\mu f(...)\right)$
Strongly coupled: classical supergravity requires $\lambda \gg 1$ but $\lambda N_c^{-1} = g_{YM} \ll 1$ so

$$\frac{1}{T N_c^{2/3}} \ll \frac{\eta}{sT} \quad \left(\quad or \quad \frac{1}{\sqrt{\lambda} T}\right) \ll L_{macro}$$

QGP: $N_c = 3 \ll \infty$,so $l_{micro} \sim \frac{\eta}{sT}$. Cold atoms: $l_{micro} \sim n^{-1/3} > \frac{\eta}{sT}$?

Why is $l_{micro} \ll l_{mfp}$ necessary? microscopic fluctuations (which have nothing to do with viscosity) will drive fluid evolution. $\Delta\rho/\rho \sim C_V^{-1} \sim N_c^{-2}$



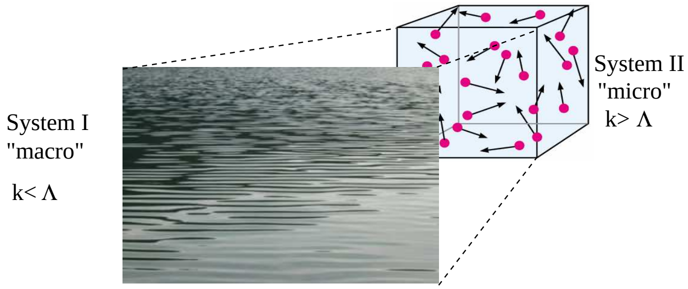


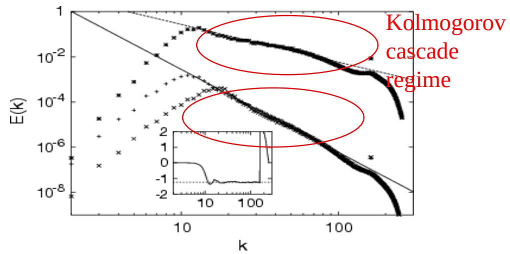
A classical low-viscosity fluid is turbulent. Typically, low-k modes cascade into higher and higher k modes In a non-relativistic incompressible fluid

$$\eta/(sT) \ll L_{eddy} \ll L_{boundary} \quad , \quad E(k) \sim \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3}$$

For a classical ideal fluid, no limit! since $\lim_{\delta\rho\to 0, k\to\infty} \delta E(k) \sim \delta\rho k c_s \to 0$ but quantum $E \geq k$ so energy conservation has to cap cascade.

More fundamentally: take stationary slab of fluid at local equilibrium.

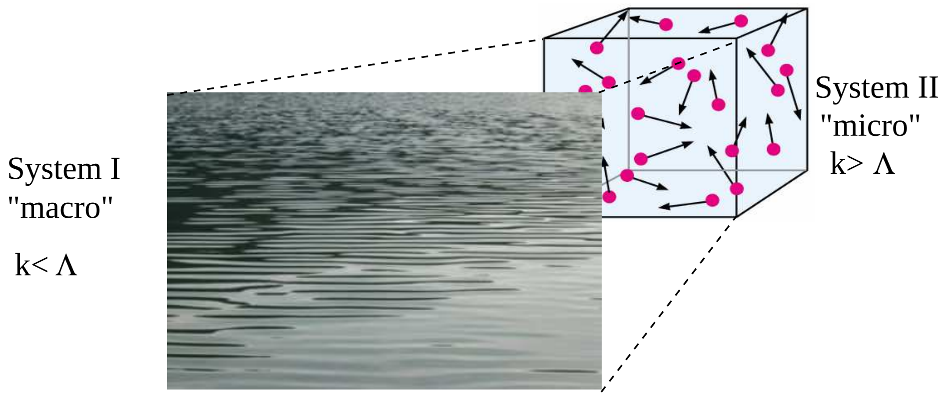


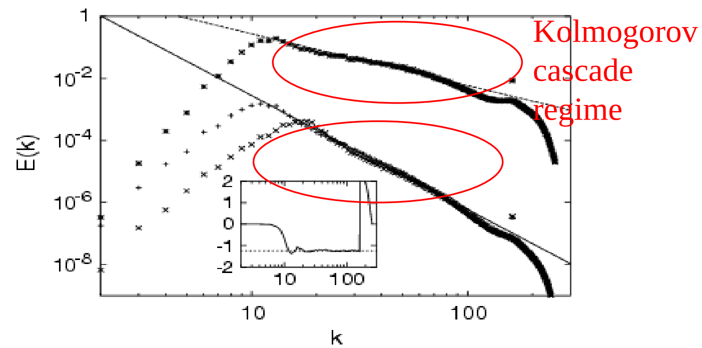


Statistical mechanics: This is a system in global equilibrium, described by a partition function $Z(T,V,\mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\langle (\Delta E)^2 \rangle$ etc. in terms of conserved charges. All microstates equally likely, which leads to preferred macrostates!

Fluid dynamics: This is the state of a field in local equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the Euler/N-S equations. What are micro/macrostates?

More fundamentally: take stationary slab of fluid at local equilibrium.





To what extent are these two pictures the same?

- Global equilibrium is also local equilibrium, <u>if</u> you <u>forget</u> fluctuations

- Dissipation scale in local equilibrium $\eta/(Ts)$, global equilibration timescale $(Ts)/\eta$

Some insight from maths

Millenium problem: existence and smoothness of the Navier-Stokes equations

Important tool are "weak solutions" , similar to what we call "coarse-graining".

$$F\left(\frac{d}{dx}, f(x)\right) = 0 \Rightarrow F\left(\int \frac{d}{dx}\phi(x)..., f(x)\right) = 0$$

$\phi(x)$ "test function", similar to coarse-graining!

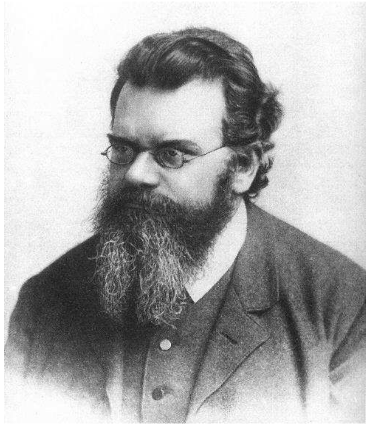
Existance of Wild/Nightmare solutions and non-uniqueness of weak solutions shows this tension is non-trivial, coarse-graining “dangerous”

I am a physicist so I care little about the ”existence of ethernal solutions” to an approximate equation, Turbulent regime and microscopic local equilibria need to be consistent

Thermal fluctuations could both ”stabilize” hydrodynamics and ”accellerate” local thermalization
But where do microstates,”local” microstates fit here?

the battle

of the entropies



Boltzmann entropy is usually a property of the "DoF", and is "kinetic" subject to the H-theorem which is really a consequence of the not-so-justified molecular chaos assumption. Gibbsian entropy is the log of the area of phase space, and is justified from coarse-graining and ergodicity , but hard to define it in non-equilibrium . The two are different even in equilibrium, with interactions! Note, Von Neumann $\langle ln\hat{\rho}\rangle$ Gibbsian

Every statistical theory needs a "state space" and an "evolution dynamics"
The ingredients

State space:Zubarev hydrodynamics Mixes micro and macro DoFs

Dynamics: Crooks fluctuation theorem provides the dynamics via a definition of $\Pi_{\mu\nu}$ from fluctuations

$\hat{T}^{\mu\nu}$ is an operator, so any decomposition, such as $\hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$ must be too!

Zubarev partition function for local equilibrium: think of Eigenstate thermalization...

Let us generalize the GC ensemble to a co-moving frame $E/T \to \beta_\mu T^\mu_\nu$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Z is a partition function with a field of Lagrange multiplies β_μ , with microscopic and quantum fluctuations included.

Effective action from $\ln[Z]$. Correction to Lagrangian picture?

All normalizations diverge but hey, it's QFT! (Later we resolve this!)

This is perfect global equilibrium. What about imperfect local?

- Dynamics is not clear. Becattini et al, 1902.01089: Gradient expansion in β_μ . Reproduces Euler and Navier-Stokes, but...
  - 2nd order Gradient expansion (Navier stokes) non-causal perhaps...
  - Use Israel-Stewart, $\Pi_{\mu\nu}$ arbitrary perhaps...
  - Foliation $d\Sigma_\mu$ arbitrary but not clear how to link to Arbitrary $\Pi_{\mu\nu}$

- What about fluctuations? Coarse-graining and fluctuations mix? How does one truncate?

An operator formulation

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}_{\mu\nu}$$

and $\hat{T}_0^{\mu\nu}$ truly in equilibrium!

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

describes all cumulants and probabilities

$$\langle T_0^{\mu\nu}(x_1) T_0^{\mu\nu}(x_2) ... T_0^{\mu\nu}(x_n) \rangle = \prod_i \frac{\delta^n}{\delta \beta_\mu(x_i)} \ln Z$$

and also the full energy-momentum tensor

$$\langle T^{\mu\nu}(x_1)T^{\mu\nu}(x_2)...T^{\mu\nu}(x_n)\rangle = \prod_i \frac{\delta^n}{\delta g_{\mu\nu}(x_i)} \ln Z$$

What this means

- Equilibrium at "probabilistic" level

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$$

- KMS Condition obeyed by "part of density matrix" in equilibrium, "expand" around that! An operator constrained by KMS condition is still an operator! $\equiv$ time dependence in interaction picture

Does this make sense

$$\hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu} \qquad , \qquad \hat{\rho}_{T_{\mu\nu}} = \frac{\hat{\rho}_{T_0} + \hat{\rho}_{\Pi_0}}{\mathrm{Tr}\left(\hat{\rho}_{T_0} + \hat{\rho}_{\Pi_0}\right)} \simeq \hat{\rho}_{T_0}\left(1 + \delta\hat{\rho}\right)$$

For <u>any</u> flow field β_μ and <u>lagrangian</u> we can <u>define</u>

$$Z_{T_0}(J(y)) = \int \mathcal{D}\phi \exp\left[-\int_0^{T^{-1}(x_i^\mu)} d\tau' \int d^3x \left(L(\phi) + J(y)\phi\right)\right] \propto$$

$$\propto \exp\left[-\beta^0 \hat{T}_{00}\right]\Big|_{\beta_\mu = (T^{-1}(x,t),\vec{0})}$$

E.g. Nishioka, 1801.10352 $\langle x|\,\rho\,|x'\rangle =$

$$= \frac{1}{Z}\int_{\tau=-\infty}^{\tau=\infty}\int\left[\mathcal{D}\phi,\mathcal{D}y(\tau)\,\mathcal{D}y'(\tau)\right]\,e^{-iS(\phi y,y')}\cdot\underbrace{\delta\left[y(0^+)-x'\right]\,\delta\left[y'(0^-)-x\right]}_{\frac{\delta J_i(y(0^+))}{\delta J_i(x')}\frac{\delta J_j(y(0^-))}{\delta J_j(x)}}$$

$$\Rightarrow \frac{\delta^2}{\delta J_i(x)\delta J_j(x')}\ln\left[Z_{T_0}(T^{\mu\nu},J)\times Z_{\Pi}(J)\right]_{J=J_1(x)+J_2(x')}$$

$J_1(x)+J_2(x')$ chosen to respect Matsubara conditions!

Any ρ can be separated like this for <u>any</u> β_μ . The question is, is this a good approximation? "Close enough to equilibrium"

The source J related to the smearing in "weak solutions". Pure maths angle?

Entropy/Deviations from equilibrium

- In quantum mechanics Entropy function of density matrix

$$s = Tr(\hat{\rho} \ln \hat{\rho}) = \frac{d}{dT}(T \ln Z)$$

Conserved in quantum evolution, not coarse-graining/gradient expansion

- In IS entropy function of the dissipative part of E-M tensor

$$n^{\nu}\partial_{\nu}(su^{\mu}) = n^{\mu}\frac{\Pi^{\alpha\beta}}{T}\partial_{\alpha}\beta_{\beta} \quad , \quad \geq 0$$

$n_{\mu} = d\Sigma_{\mu}/|d\Sigma_{\mu}|, \Pi_{\mu\nu}$ arbitrary. How to combine coarse-graining? if vorticity non-zero $n_{\mu}u^{\mu} \neq 0$

What about fluctuations

$$n^\nu \partial_\nu \left(s u^\mu \right) = n^\mu \frac{\Pi^{\alpha\beta}}{T} \partial_\alpha \beta_\beta \qquad , \qquad \geq 0$$

- If n_μ arbitrary cannot be true for "any" choice
- 2nd law is true for "averages" anyways, <u>sometimes</u> entropy can decrease

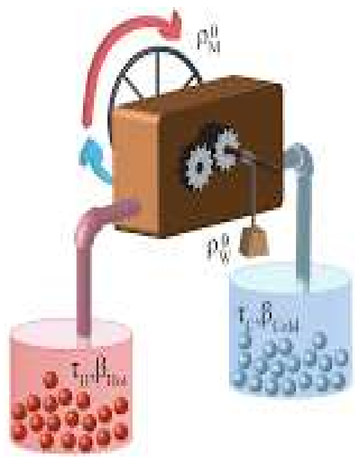
We need a fluctuating formulation!

- "Statistical" (probability depends on "local microstates")
- Dynamics with fluctuations, time evolution of β_μ distribution

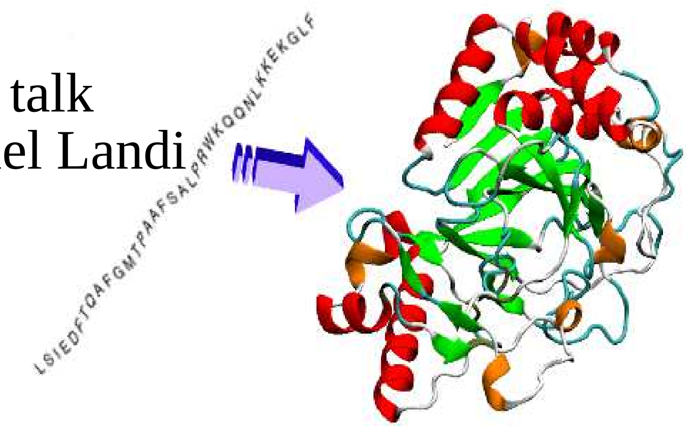
So we need

- a similarly probabilistic definition of $\hat{\Pi}^{\mu\nu} = \hat{T}^{\mu\nu} - \hat{T}_0^{\mu\nu}$ as an <u>operator</u>!!
- Probabilistic dynamics, to update $\hat{\Pi}_{\mu\nu}, \hat{T}_{\mu\nu}$!

Crooks fluctuation theorem!



From talk
Gabriel Landi



Relates fluctuations, entropy in <u>small</u> <u>fluctuating</u> systems (Nano,proteins)

Crooks fluctuation theorem!

$$P(W)/P(-W) = \exp\left[\Delta S\right]$$

P(W) Probability of a system doing some work in its usual thermal evolution

P(-W) Probability of the same system "running in reverse" and decreasing entropy due to a thermal fluctuation

ΔS Entropy produced by $P(W)$

Looks obvious but...

Is valid for systems very far from equilibrium (nano-machines, protein folding and so on)

Proven for Markovian processes and fluctuating systems in contact with thermal bath

Leads to irreducible fluctuation/dissipation: TUR (more later!)

Applying it to locally equilibrium systems within Zubarev's formalism is straight-forward . Since ratios of probabilities, divergences are resolved!

How is Crooks theorem useful for what we did? Guarnieri et al, arXiv:1901.10428 (PRX) derive Thermodynamic uncertainity relations from

$$\hat{\rho}_{ness} \simeq \hat{\rho}_{les}(\lambda) e^{\hat{\Sigma}} \frac{Z_{les}}{Z_{ness}} \quad , \quad \hat{\rho}_{les} = \frac{1}{Z_{les}} \exp\left[-\frac{\hat{H}}{T} \right]$$

$\hat{\rho}_{les}$ is Zubarev operator while Σ is calculated with a Kubo-like formula

$$\hat{\Sigma} = \delta_\beta \Delta \hat{H}_+ \quad , \quad \hat{H}_+ = \lim_{\epsilon \to 0^+} \epsilon \int dt e^{\epsilon t} e^{-\hat{H} t} \Delta \hat{H} e^{\hat{H} t}$$

Relies on

$$\lim_{w \to 0} \left\langle \left[\hat{\Sigma}, \hat{H} \right] \right\rangle \to 0 \equiv \lim_{t \to \infty} \left\langle \left[\hat{\Sigma(t)}, \hat{H}(0) \right] \right\rangle \to 0$$

This "infinite" is "small" w.r.t. hydro gradients. $\equiv$ Markovian as in Hydro with $l_{mfp} \to \partial$ but with operators$\to$ carries all fluctuations with it!

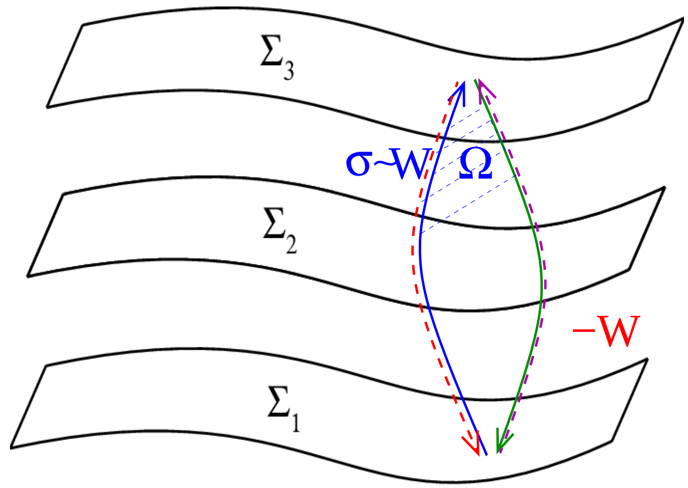
$$P(W)/P(-W) = \exp[\Delta S] \quad \mathrm{Vs} \quad S_{eff} = \ln Z$$

KMS condition reduces the functional integral to a Metropolis type weighting, $\equiv$ periodic time at rest with β_μ

Markovian systems exhibit Crooks theorem, two adjacent cells interaction outcome probability proportional to number of ways of reaching outcome . The normalization divergence is resolved since ratios of probabilities are used . "instant decoherence/thermalization" within each step

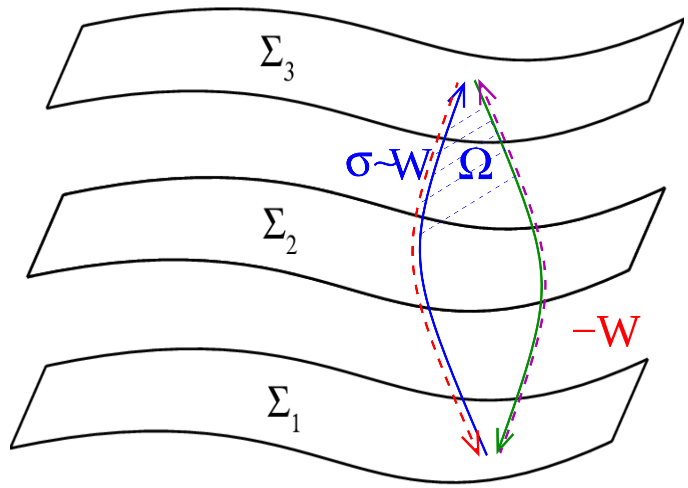
Relationship to gradient expansion similar to relationship between Wilson loop coarse-graining (Jarzynski's theorem, used on lattice ,Caselle et al, 1604.05544) with hadronic EFTs

Applying Crooks theorem to Zubarev hydrodynamics: Stokes theorem



$$-\int_{\Sigma(\tau_0)} \partial\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) = -\int_{\Sigma(\tau')} \partial\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) + \int_\Omega \partial\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu\right),$$

true for "any" fluctuating configuration.



Let us now invert one foliation so it goes "backwards in time" assuming Crooks theorem means

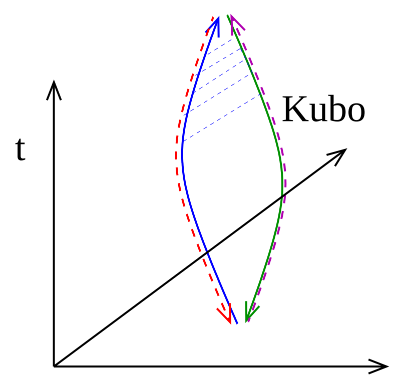
$$\frac{\exp\left[-\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]}{\exp\left[-\int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]} = \exp\left[\frac{1}{2}\int_\Omega d\Omega_\mu^\mu \left[\frac{\hat{\Pi}^{\alpha\beta}}{T}\right] \partial_\beta \beta_\alpha\right]$$

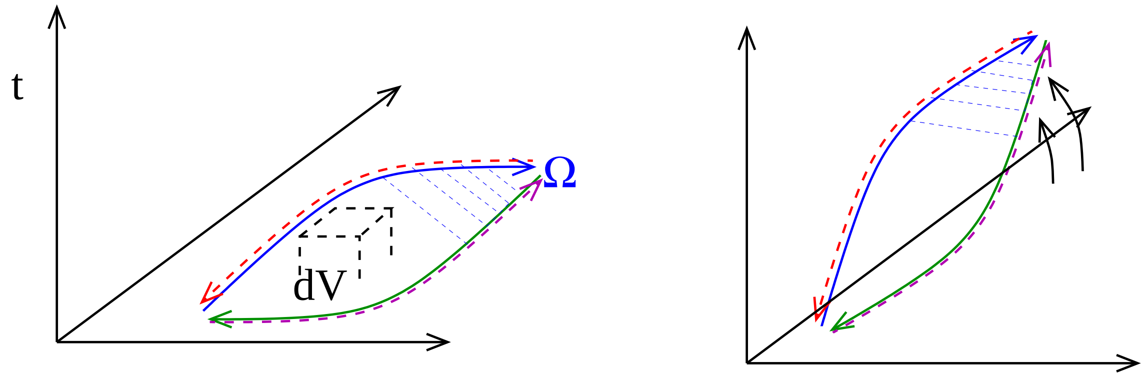
Small loop limit $\left\langle \exp\left[\oint d\Sigma_\mu \omega^{\mu\nu} \beta^\alpha \hat{T}_{\alpha\nu} \right] \right\rangle = \left\langle \exp\left[\int \frac{1}{2} d\Sigma_\mu \beta^\mu \hat{\Pi}^{\alpha\beta} \partial_\alpha \beta_\beta \right] \right\rangle$

A non-perturbative operator equation,divergences cancel out...

$$\left.\frac{\hat{\Pi}^{\mu\nu}}{T}\right|_\sigma = \left(\frac{1}{\partial_\mu \beta_\nu}\right) \frac{\delta}{\delta\sigma} \left[\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu} - \int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu} \right]$$

Note that a time-like contour produces a Kubo-formula

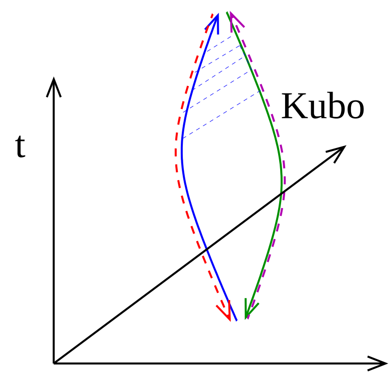




A sanity check: For a an equilibrium spacelike $d\Sigma_\mu = (dV, \vec{0})$ (left-panel) we recover Boltzmann's

$$\Pi^{\mu\nu} \Rightarrow \Delta S = \frac{dQ}{T} = \ln\left(\frac{N_1}{N_2}\right)$$

A sanity check



When $\eta \to 0$ *and* $s^{-1/3} \to 0$ (the first two terms in the hierarchy), Crooks fluctuation theorem gives $P(W) \to 1$ $P(-W) \to 0$ $\Delta S \to \infty$ so Crooks theorem reduces to δ-functions of the entropy current

$$\delta\left(d\Sigma_\mu\left(su^\mu\right)\right) \Rightarrow n^\mu \partial_\mu\left(su^\mu\right) = 0$$

We therefore recover conservation equations for the entropy current, a.k.a. ideal hydro

Crooks theorem: thermodynamic uncertainity relations

Andr M. Timpanaro, Giacomo Guarnieri, John Goold, and Gabriel T. Landi
Phys. Rev. Lett. 123, 090604

$$\frac{\left\langle(\Delta Q)^2\right\rangle}{\langle Q\rangle^2} \geq \frac{2}{\Delta S(W)}$$

Valid locally in time!

$$\frac{d}{d\tau}\Delta S \geq \frac{1}{2}\frac{d}{d\tau}\frac{\langle Q\rangle^2}{\langle(\Delta Q)^2\rangle}$$

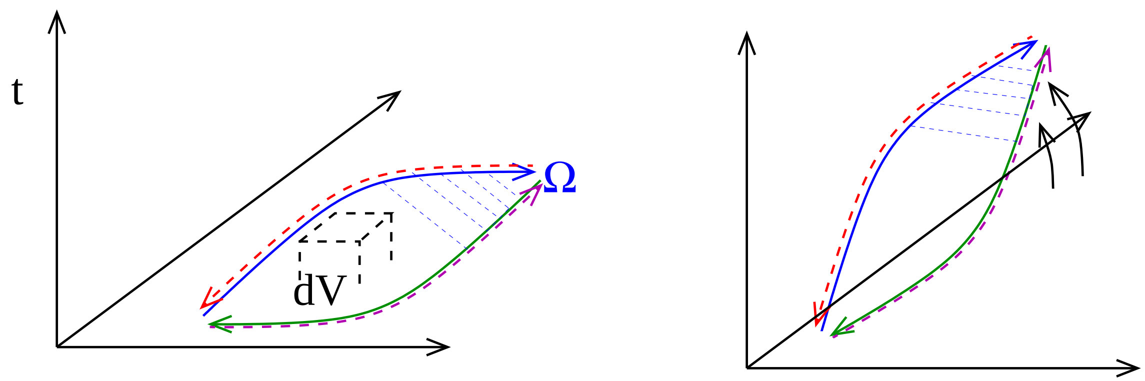
Relates thermal fluctuations and dissipation, producing an irreducible uncertainity. Non-dissipative nano-engines fluctuate like crazy, produces "dissipation" anyway

COnsequences: Hydro-TUR? Separate flow into potential and vortical part

$$\beta_\mu = \partial_\mu \phi + \zeta_\mu \quad , \quad n_\mu \to T \partial_\mu \phi \quad , \quad \omega_{\mu\nu} = g_{\mu\nu}|_{comoving}$$

A likely TUR is

$$\frac{\langle [T_{\mu\gamma}, T_\nu^\gamma] \rangle}{\langle T^{\mu\nu} \rangle^2} \geq \frac{\mathcal{C} \epsilon_{\mu\gamma\kappa} \langle T^{\gamma\kappa} \rangle \beta^\mu}{\Pi^{\alpha\beta} \partial_\beta \zeta_\alpha} \quad , \quad \mathcal{C} \sim \mathcal{O}(1)$$

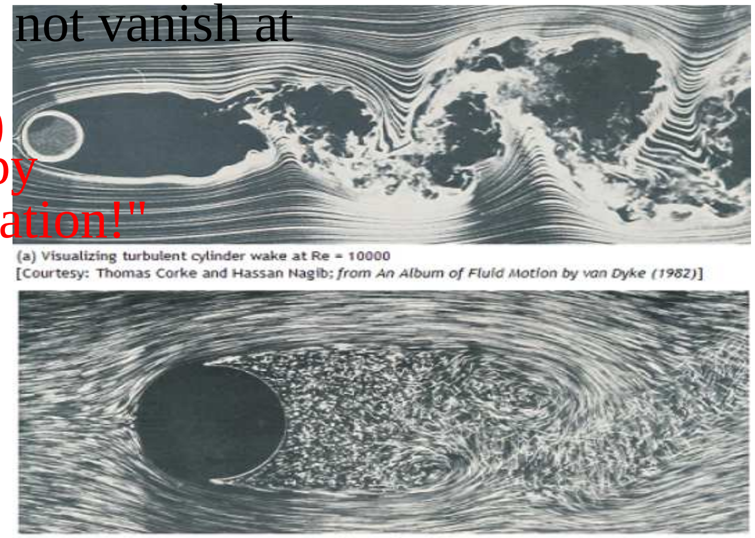


Deform the equilibrium contour and get Kubo formula! (right panel)

$$\mathcal{C} = \lim_{w \to 0} \frac{\mathrm{Re}\left[F(w)\right]}{\mathrm{Im}\left[F(w)\right]} \quad , \quad F(w) = \int d^3x dt \left\langle T^{xy}(x) T^{xy}(0) \right\rangle e^{i(kx - wt)}$$

−dissipation does not vanish at zero viscosity

Vlad Vicol (talk)
"will be proven by
a different generation!"



(a) Visualizing turbulent cylinder wake at Re = 10000
[Courtesy: Thomas Corke and Hassan Nagib; *from An Album of Fluid Motion by van Dyke (1982)*]

(b) a closer look at Re = 2000 - patterns are identical as in (a)
[Courtesy: ONERA pic. Werle & Gallon (1972) *from An Album of Fluid Motion by van Dyke (1982)*]

$$\frac{\langle [T_{\mu\gamma}, T_{\nu}^{\gamma}] \rangle}{\langle T^{\mu\nu} \rangle^2} \geq \frac{\mathcal{O}(1)\, \epsilon_{\mu\gamma\kappa} \langle T^{\gamma\kappa} \rangle \beta^{\mu}}{\Pi^{\alpha\beta} \partial_\beta \zeta_\alpha}$$

Fluctuations+Low viscosity $\Rightarrow$ Turbulence $\Rightarrow$ high vorticity $\Rightarrow$ dissipation!
(usually mathematicians consider incompressible fluids, non-relativistic!)

Towards equations: Gravitational Ward identity!

$$\partial^{\alpha}\left\{\left\langle\left[\hat{T}_{\mu\nu}(x),\hat{T}_{\alpha\beta}(x')\right]\right\rangle -\right.$$

$$\left. -\delta(x-x')\left(g_{\beta\mu}\left\langle\hat{T}_{\alpha\nu}(x')\right\rangle + g_{\beta\nu}\left\langle\hat{T}_{\alpha\mu}(x')\right\rangle - g_{\beta\alpha}\left\langle\hat{T}_{\mu\nu}(x')\right\rangle\right)\right\} = 0$$

Small change in $T_{\mu\nu}$ related to infinitesimal shift! Conservation of momentum!

Can be used to fix one component of $\beta_{\mu} = u_{\mu}/T$, so $u_{\mu}u^{\mu} = -1$ and $(\beta_{\mu}\beta^{\mu})^{-1/2} = T$ weights $\hat{\Pi}^{\mu\nu}$ in a way that conserves $\hat{\Pi}^{\mu\nu} + \hat{T}_0^{\mu\nu}$

Putting everything together: Dynamics at Z level

$$\langle T_{\mu\nu}\rangle = \frac{2}{\sqrt{-g}}\frac{\delta \ln Z}{\delta g^{\mu\nu}} = \langle T_0\rangle^{\mu\nu} + \Pi^{\mu\nu}$$

$$\langle T_0^{\mu\nu}\rangle = \frac{\delta^2 \ln Z}{\delta \beta_\mu d n_\nu} \qquad , \qquad \langle \Pi^{\mu\nu}\rangle = \frac{1}{\partial_\mu \beta_\nu}\partial_\gamma \frac{d}{d\ln(\beta_\alpha\beta^\alpha)}\left[\beta^\gamma \ln Z\right]$$

$$\partial_\alpha\left[\frac{2}{\sqrt{-g}}\frac{\delta^2 \ln Z}{\delta g_{\mu\nu}\delta g_{\alpha\beta}} - \delta(x-x')\frac{2}{\sqrt{-g}}\left(g_{\beta\mu}\frac{\delta \ln Z}{\delta g_{\alpha\nu}} + g_{\beta\nu}\frac{\delta \ln Z}{\delta g_{\alpha\mu}} - g_{\beta\alpha}\frac{\delta \ln Z}{\delta g_{\nu\mu}}\right)\right] = 0$$

and, finally, Crook's theorem

$$\frac{\delta^2}{\delta g^{\mu\nu}\delta g^{\alpha\beta}}\ln Z = \frac{\sqrt{-g}}{2}\frac{\beta_\kappa}{2\omega^{\mu\nu}\beta^\alpha}\partial_\beta n^\kappa \partial_\gamma \frac{d}{d\ln(\beta_\alpha\beta^\alpha)}\left[\beta^\gamma \ln Z\right]$$

A numerical formulation

Define a field β_μ field and n_μ

Generate an ensemble of

$$\ln Z|_{t+dt} = \int \mathcal{D} g_{\mu\nu}(x) T^{\mu\nu}|_{t+dt} \quad , \quad \beta_\mu|_{t+dt} = \frac{\delta \ln Z|_{t+dt}}{\delta T_{\mu\nu}} n_\nu$$

According to a Metropolis algorithm ran via Crooks theorem

Reconstruct the new β and $\Pi_{\mu\nu}$. The Ward identity will make sure $\beta_\mu \beta^\mu = -1/T^2$

Computationally intensive (an ensemble at every timestep), but who knows?

A semiclassical limit?

$$\partial_\mu \left\langle \hat{T}^{\mu\nu} \right\rangle = 0 \quad , \quad \partial_\mu \left\langle \hat{T}_0^{\mu\nu} \right\rangle = -\partial_\mu \left\langle \hat{\Pi}^{\mu\nu} \right\rangle$$

Integrating by parts the second term over a time scale of many $\Delta_{\mu\nu}$ gives, in a frame comoving with $d\Sigma_\mu$

$$\int_0^\tau d\tau' \left\langle \hat{\Pi}_{\mu\nu} \right\rangle \partial^\mu \beta^\nu \sim \beta^\mu \partial_\mu \left\langle \hat{\Pi}_{\mu\nu} \right\rangle + \left\langle \hat{\Pi}_{\mu\nu} \right\rangle = F(\partial^{n\geq 1}\beta_\mu, ...)$$

where $F(\beta_\mu)$ is independent of $\Pi_{\mu\nu}$. (Because local entropy is maximized at vanishing viscosity $F()$ depends on gradients. Israel-Stewart

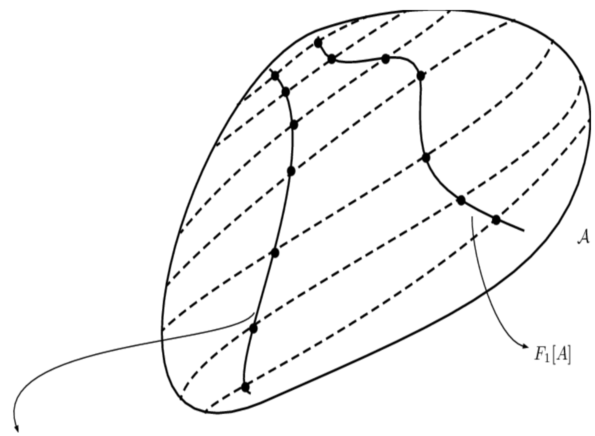
However , results of, e.g., Gavassino 2006.09843 and Shokri 2002.04719 suggest that fluctuations with decreasing entropy have a role at first order in gradient!

So could fluctuations help thermalize? A key insight is redundances

Some qualitative developments: $T_0^{\mu\nu}, \Pi^{\mu\nu}, u^\mu$ are not actually experimental observables! Only total energy momentum tensor

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$$

and its correlators are! Changing $d\Sigma_\mu$ in Zubarev $\equiv$ changing $\Pi^{\mu\nu}, T_0^{\mu\nu}$!



Analogy to choosing a gauge in gauge theory?

This is relevant for current hydrodynamic research

Causal relativistic hydrodynamics still contentious, with many definitions

Israel-Stewart Relaxing $\Pi_{\mu\nu}$.

Causal, but up to 9 additional DoFs (not counting conserved charges), blow-up possible (M.Disconzi, 2008.03841). $\Pi_{\mu\nu}$ "evolving" microstates!

BDNK,earlier Hiscock,Lindblom,Geroch,... $\Pi_{\mu\nu} \sim \partial u$ At a price of arbitrary (up to causality constaints) u_μ . If you care about statistical mechanics, price is steep! "special" time foliation from ergodic hypothesis/Poncaire cycles!

For phenomenology because of conservation laws "any" $\partial_\mu T^{\mu\nu}$ "can be integrated" but lack of link with equilibration and multiple definitions of "near-equilibrium" problematic. Could these be just "Gauge" choices?

What is a gauge theory,exactly?

$$\mathcal{Z} = \int \mathcal{D}A^\mu \exp\left[S[F_{\mu\nu}\right] \equiv \int \mathcal{D}A_1^\mu \mathcal{D}A_2^\mu \exp\left[S[A_1^\mu\right]$$

$A_{1,2}^\mu$ can be separated since physics sensitive to derivatives of $\ln \mathcal{Z}$

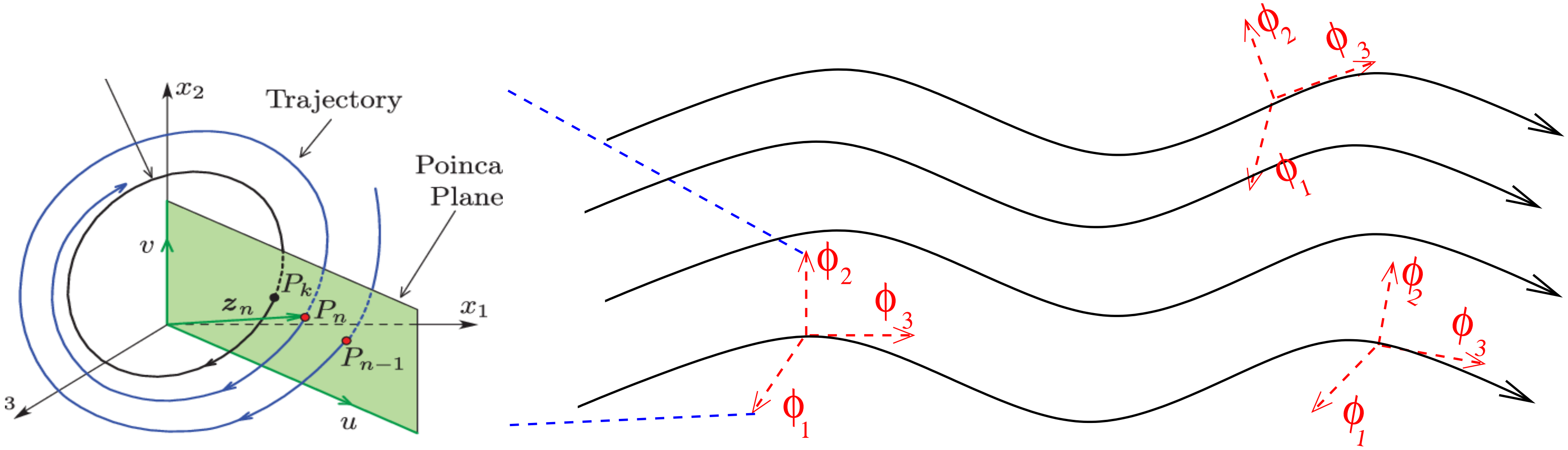
$$\ln \mathcal{Z} = \Lambda + \ln \mathcal{Z}_G \quad , \quad Z_G = \int \mathcal{D}\mathcal{A}^\mu \delta\left(G(A^\mu)\right) \exp\left[S(A_\mu)\right]$$

Ghosts come from expanding $\delta(...)$ term. In Zubarev

$$Z = \int \mathcal{D}\phi \quad , \quad "S" = d\Sigma_\nu \beta_\mu T^{\mu\nu}$$

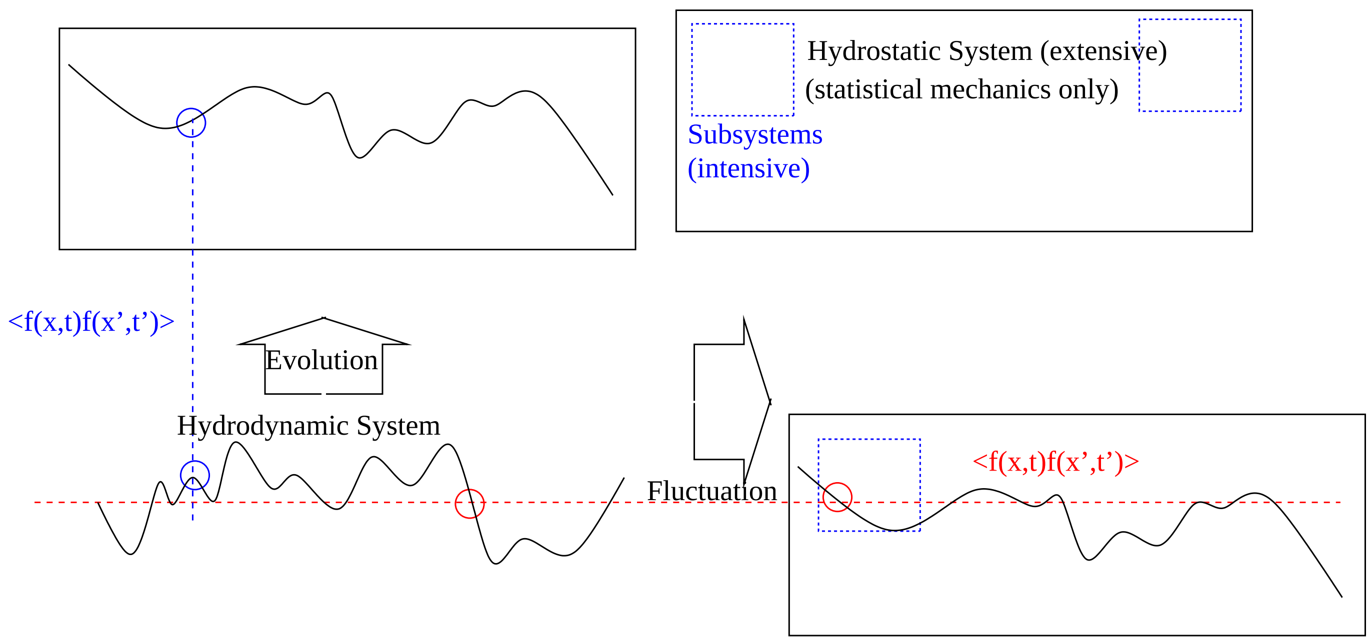
Multiple $T_{\mu\nu}(\phi) \rightarrow$ Gauge-like configuration . Related to Phase space fluctuations of ϕ

How to make physics fully "gauge"-invariant? Ergodicity/Poncaire cycles meet relativity slightly away from equilibrium!

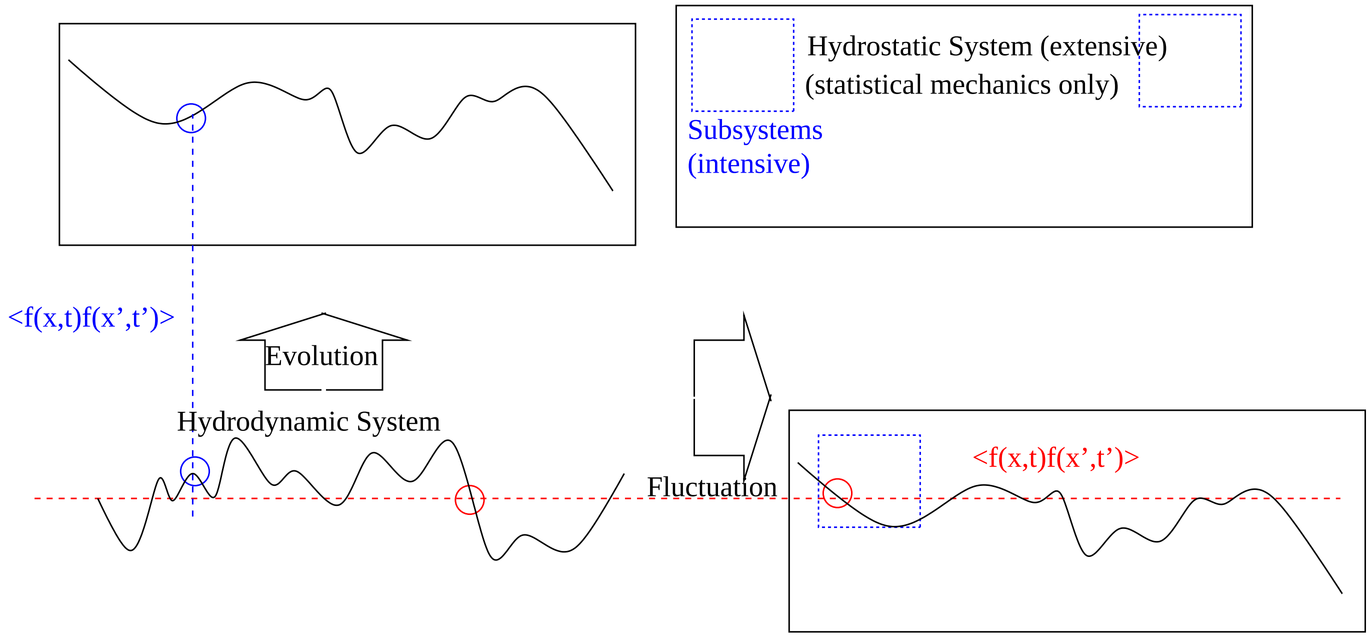


Gibbs entropy level+relativity : Lack of equilibrium is equivalent to "loss of phase" of Poncaire cycles. one can see a slightly out of equilibrium cell <u>either</u> as a "mismatched u_μ" (fluctuation) or as lack of genuine equilibrium (dissipation)

How to make physics fully "gauge"-invariant?



Fluctuation-dissipation at the cell level could do it! We don't know if a "step" is fluctuation ($T_0^{\mu\nu}$ or evolution ($\Pi_{\mu\nu}$)-driven!



But in hydro $T_0^{\mu\nu}, \Pi_{\mu\nu}$ treated very differently! "Sound-wave"
$u \sim \exp[ik_\mu x^\mu]$ or "non-hydrodynamic Israel-Stewart mode?"
$D\Pi_{\mu\nu} + \Pi_{\mu\nu} = \partial u$
Only in EFT $1/T \ll l_{mfp}$ they are truly different!

Infinitesimal transformation $dM_{\mu\nu}$ such that $dM_{\mu\nu}(x)\frac{\delta \ln \mathcal{Z}_E[\beta_\mu]}{dg^{\alpha\mu}(x)} = 0$

Change in microscopic fluctuation $\ln \mathcal{Z} \to \ln \mathcal{Z} + d \ln \mathcal{Z}$

$$d \ln \mathcal{Z} = \sum_{N=0}^{\infty} \int \prod_{j=1}^{N} d^4 p_j \delta \left(E_N(p_1, ...p_j) - \sum_j p_j^0 \right) \sqrt{|dM|} \exp \left(-\frac{dM_{0\mu} p^\mu}{T} \right)$$

Change in macroscopic dissipative term

$$\Pi_{\mu\nu} \to \Pi_{\alpha\gamma} \left(g_\mu^\alpha g_\nu^\gamma - g_\mu^\alpha dM_\nu^\gamma - g_\nu^\gamma dM_\mu^\alpha \right) \quad , \quad u_\mu \to u_\alpha \left(g_\mu^\alpha - dM_\mu^\alpha \right)$$

For $1/T \ll l_{mfp}$ probability of this <u>vanishes</u>, but for $1/T \sim l_{mfp}$ many "similar" probabilities!

The "gauge-symmetry" in practice

Generally $dM_{\mu\nu} = \Lambda^{-1}_{\alpha\mu} dU^{\alpha\beta} \Lambda_{\beta\mu}$

$$d\left[\ln \Pi_{\alpha\beta}\right] \Lambda^{\alpha\mu} \left(\Lambda^{\beta\nu}\right)^{-1} = \eta^{\mu\nu} d\mathcal{A} + \sum_{I=1,3} \left(d\alpha_I \hat{J}^{\mu\nu}_I + d\beta_I \hat{K}^{\mu\nu}_I\right)$$

$$K_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

which move components from $\Pi_{\mu\nu}$ to Q_μ as well as $K_{1,2,3}$

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, J_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Towards hydrodynamic <u>Gibbsian</u> entropy definition !

$$\int \mathcal{D}\phi e^{-S(\phi)} \underbrace{\rightarrow}_{coarse-grain} \int \mathcal{D}\alpha_{I=1,2,3}\mathcal{D}\beta_{I=1,2,3}\ \mathcal{D}\left[\mathcal{A}, e, p, u_\mu, \Pi_{\mu\nu}\right]$$

$$\delta\left(M_{\alpha\beta}\left[\mathcal{A}, \alpha_I, \beta_I\right] T^{\alpha\mu}\right)$$

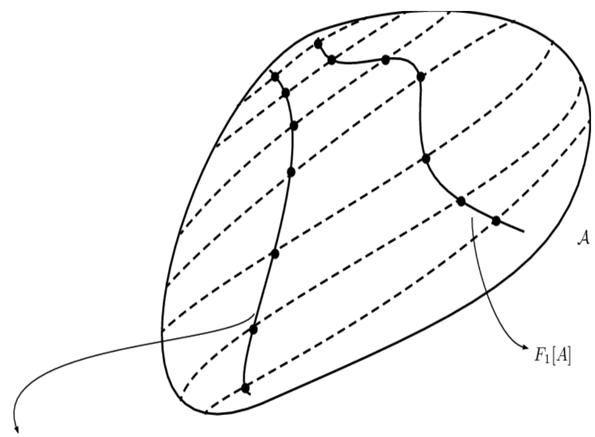
rotate "Gradient expansion" in $1/T, l_{mfp}$ parameter space.
Away from Boltzmann equation regime, $f(x,p) \rightarrow$ Functional

lagrangian , $\ln \mathcal{Z}$ subject to $\delta(...)$ constraint.

Causality also defined via <u>correlator</u> $[T_{\mu\nu}(x), T_{\mu\nu}(x')]$ $e, u_\mu \Pi_{\mu\nu}$ <u>could</u> be non-causal!

Cool but what about thermalization in small systems?

Initial and final state described by many equivalent trajectories



One of them could be close to an ideal-looking one. "reverse" attractor Few particles with strong interaction (Eigenstate thermalization?) correspond to many hydro like-configurations $\{u_\mu, \Pi_{\mu\nu}\}$ with fluctuations , within same Gibbs entropy class. some closer to ideal? No symmetries necessary!

Irrelevant in everyday liquids since $l_{mfp} \gg 1/T$ or AdS/CFT since $N_c \ll \infty$ but perhaps not for QGP!

Conclusions

- Linking hydrodynamics to statistical mechanics is still an open problem
  Only top-down models (Boltzmann,AdS/CFT) rather than bottom-up theory
  Is hydro universal? what are its limits of applicability? still open question

  The observation of hydro-like behavior in small systems liable to fluctuations makes this explicit!

- Crooks fluctuation theorem could provide such a link!

- redundances play crucial role in fluctuations, could mean small systems achieve "thermalization" quicker! inverse attractor!

- An obvious extension/application is...

PS: transfer of micro to macro DoFs experimentally proven!

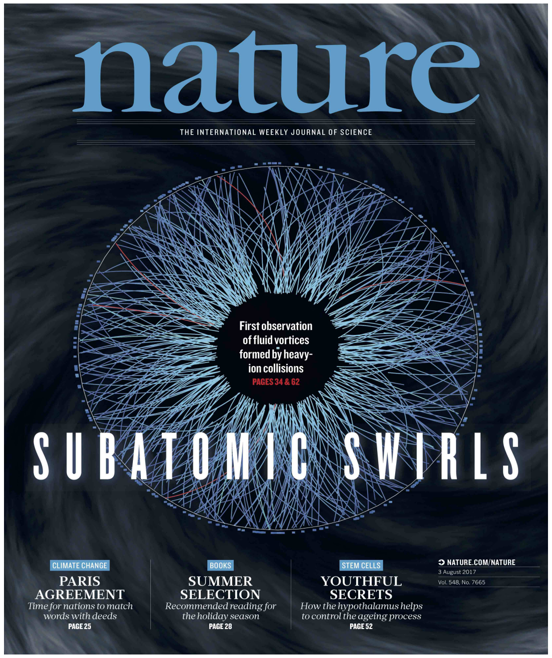
STAR collaboration

1701.06657

NATURE

August 2017

Polarization by vorticity in heavy ion collisions



Could give new talk about this, but will mention hydro with spin not developed and a lot of conceptual debates Pseudo-gauge dependence if both spin and angular momentum present in fluid? Gauge symmetry "ghosts"? GT,1810.12468 (EPJA) . redundances?

STAR collaboration

1701.06657

NATURE

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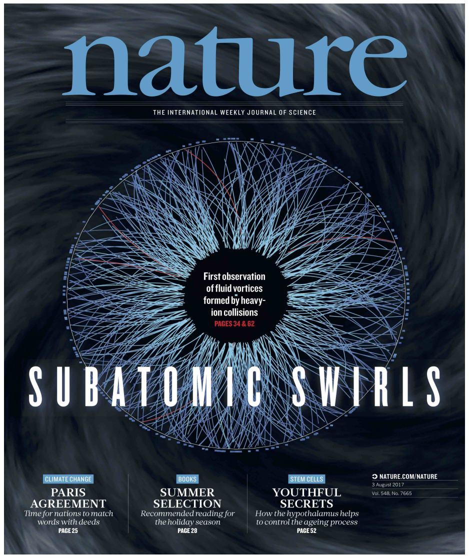
Polarization by vorticity in heavy ion collisions



Pseudo-gauge symmetries physical interpretation: T.Brauner, 1910.12224

$$x^\mu \to x^\mu + \epsilon \zeta^\mu(x) \quad , \quad \psi_a \to \psi_a + \epsilon \psi'_a \to \mathcal{L} \to \mathcal{L}$$

$\ln \mathcal{Z}$ Invariant, but $\langle O \rangle$ generally is not. Spin $\leftrightarrow$ fluctuation, need equivalent of DSE equations! $D \langle O \rangle = 0 \to D \langle O \rangle = \langle O_I O_J \rangle$

Vlasov equation contains all classical correlations, instability-ridden

Boltzmann equation "Classical UV-completion" ov Vlasov equation, first term in BBGK hyerarchy, written in terms of Wigner functions.

Finite number of particles: $f(x,p)$ not a function but a functional ($\mathcal{F}(f(x,p)) \underbrace{\rightarrow}_{Boltzmann} \delta\left(f' - f(x,p)\right)$), incorporating continuum of functions and all correlations. Perhaps solvable!

$$\frac{p^\mu}{\Lambda}\frac{\partial}{\partial x^\mu} f(x,p) = \left\langle \underbrace{\hat{C}[\tilde{W}(\tilde{f}_1,\tilde{f}_2)] - g\frac{p^\mu}{\Lambda}\hat{F}^{\mu\nu}[\tilde{f}_1,\tilde{f}_2]\frac{\delta}{\delta \tilde{f}_{1,2}}\tilde{W}\left(\tilde{f}_1,\tilde{f}_2\right)}_{How \quad many \quad A-B=0?} \right\rangle$$

The difference in collision-term redundancy-ridden!