Application of multi-particle techniques for flow analyses in CBM at FAIR

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"Workshop on analysis techniques for centrality determination and flow measurements at FAIR and NICA", 27/08/2020

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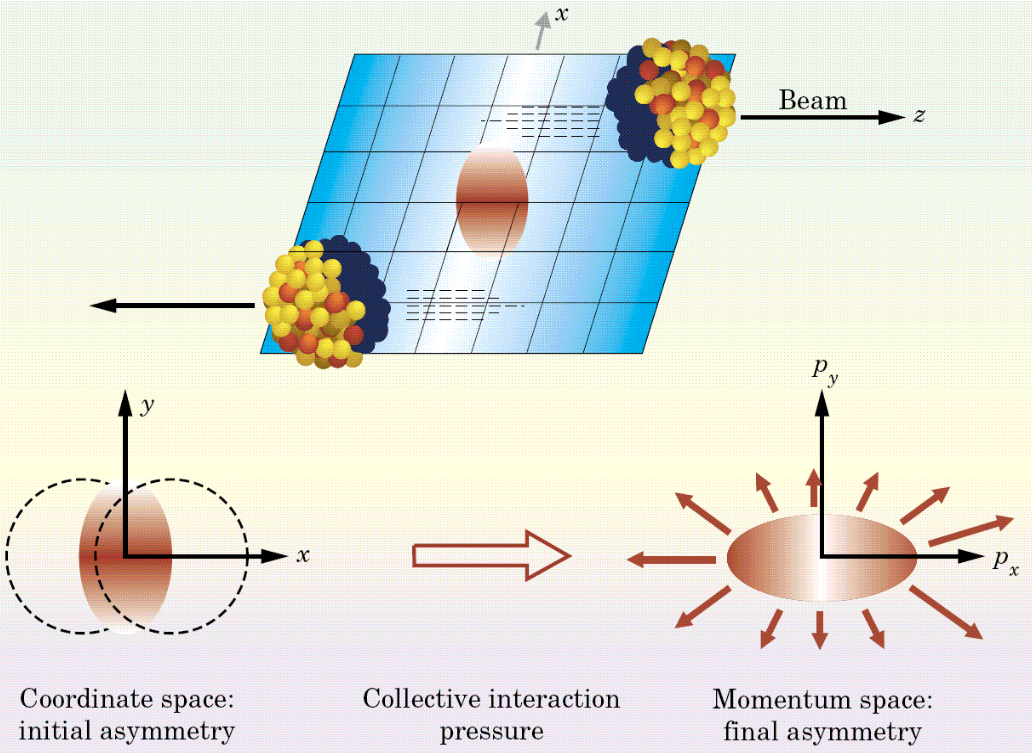
Outline

- Introduction
  - Cornerstones of flow analyses with correlation techniques
  - Critical checks for CBM

  For technical details on multi-particle correlations, see my talk from yesterday (http://indico.oris.mephi.ru/event/181/session/2/contribution/9)
- Analysis code, dataset and cuts
- Results
  - Control histograms
  - Acceptance corrections
  - Multi-particle correlations and cumulants vs. multiplicity
- Coming next

Anisotropic flow phenomenon

- Transfer of anisotropy from the initial coordinate space into the final momentum space via the thermalized medium:



- J.Y. Ollitrault, Phys. Rev. D **46** (1992) 229

Quantifying anisotropic flow with Fourier series

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- In the context of flow analysis, we use the 2nd parameterization to describe the anisotropic emission of particles in the transverse plane after heavy-ion collision:

$$f(\varphi) = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]\right]$$

- v_n : flow amplitudes
- Ψ_n : symmetry planes
- Anisotropic flow is quantified with v_n and Ψ_n
  - v_1 is directed flow
  - v_2 is elliptic flow
  - v_3 is triangular flow
  - v_4 is quadrangular flow, etc.

S. Voloshin and Y. Zhang, Z.Phys. C70 (1996) 665-672 (700+ citations!)

Cornerstones of flow analyses with correlation techniques

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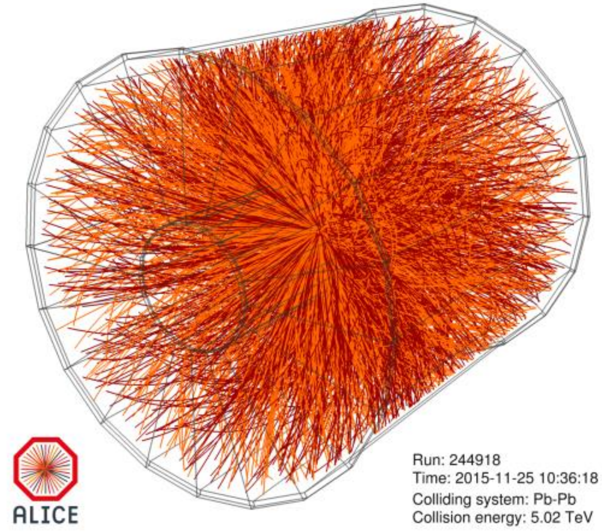
1. The analytic expression between azimuthal correlators and flow degrees of freedom

$$\langle\cos[n_1\varphi_1+\cdots+n_k\varphi_k)]\rangle = v_{n_1}\cdots v_{n_k}\cos[n_1\Psi_{n_1}+\cdots+n_k\Psi_{n_k})]$$

R. S. Bhalerao, M. Luzum and J.-Y. Ollitrault, PRC 84 034910 (2011)

- A plethora of non-trivial and independent flow observables!

We need as many independent observables as possible to describe such a complex system as heavy-ion collision



Cornerstones of flow analyses with correlation techniques

2. Specific azimuthal correlators can be selected to correspond to different flow moments (this way we can constrain the underlying p.d.f. of flow fluctuations!)

- Example:

$$\langle\cos[n(\varphi_1-\varphi_2)]\rangle = v_n^2$$
$$\langle\cos[n(\varphi_1+\varphi_2-\varphi_3-\varphi_4)]\rangle = v_n^4$$

Different flow moments carry by definition independent information about the underlying p.d.f. $f(v_n)$

$$\left\langle v_n^k \right\rangle \equiv \int v_n^k f(v_n)\, dv_n$$

Cornerstones of flow analyses with correlation techniques

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3. All multi-particle azimuthal correlators can be expressed analytically in terms of Q-vectors => self-correlations can be removed completely with a single pass over all azimuthal angles

- Example: Analytic result for 4-p correlation

$$\begin{aligned}
\langle 4\rangle &\equiv \langle\cos(n(\varphi_1+\varphi_2-\varphi_3-\varphi_4))\rangle \\
&= \frac{1}{\binom{M}{4}4!}\sum_{\substack{i,j,k,l=1\\(i\neq j\neq k\neq l)}}^{M} e^{in(\varphi_i+\varphi_j-\varphi_k-\varphi_l)} \\
&= \frac{1}{\binom{M}{4}4!}\times\left[\,|Q_n|^4+|Q_{2n}|^2-2\cdot\mathfrak{Re}\left[Q_{2n}Q_n^*Q_n^*\right]-4(M-2)\,|Q_n|^2\right. \\
&\quad \left.+2M(M-3)\right]
\end{aligned}$$

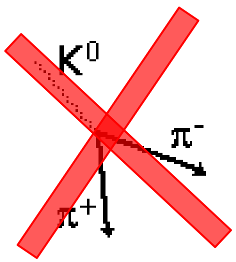
$$Q_n = \sum_{i=1}^{M} e^{in\varphi_i}$$

$$Q_{2n} = \sum_{i=1}^{M} e^{i2n\varphi_i}$$

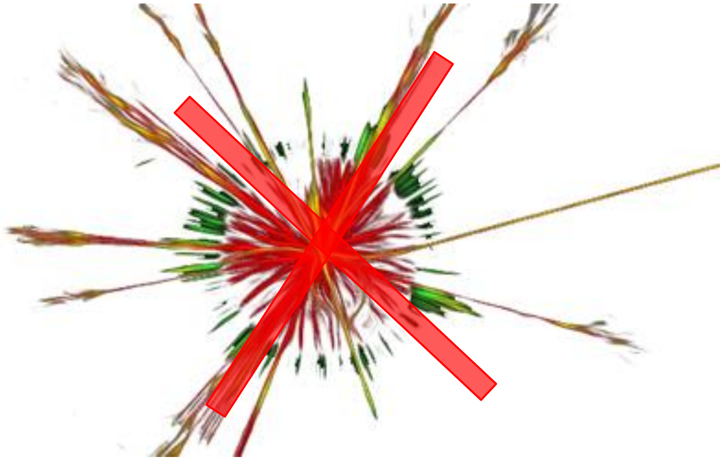
TUM

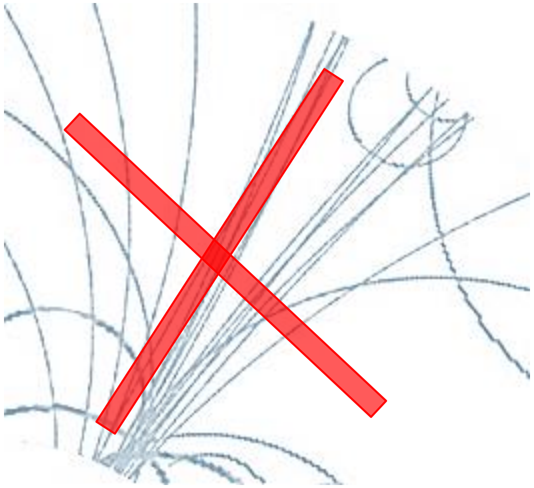
Cornerstones of flow analyses with correlation techniques

4. Multi-particle cumulants are less sensitive to nonflow than the corresponding multiparticle correlators, and therefore provide much more reliable estimates for anisotropic flow observables



resonance decays


jets


track splitting during reconstruction

References

- For the material presented in this talk, the relevant publications on multi-particle correlation techniques are:
  - *Q*-cumulants (Bilandzic *et al*, Phys. Rev. C **83** (2011) 044913)
    - First analytic expressions for few selected multiparticle correlations
  - Generic framework (Bilandzic *et al*, Phys. Rev. C **89** (2014) no.6, 064904)
    - Analytic expressions for ALL multiparticle correlations
    - Prescription to correct systematic biases due to detector inefficiencies
    - New flow observables: Symmetric Cumulants (SC)
- These two publications contain all technical details, currently being implemented for CBM

Critical check for CBM #1

- Scaling of statistical uncertainty (*N* is number of events, *M* is multiplicity, *v* is flow strength, *k* is order of correlator):

$$\sigma_v \sim \frac{1}{\sqrt{N}} \frac{1}{M^{k/2}} \frac{1}{v^{k-1}}$$

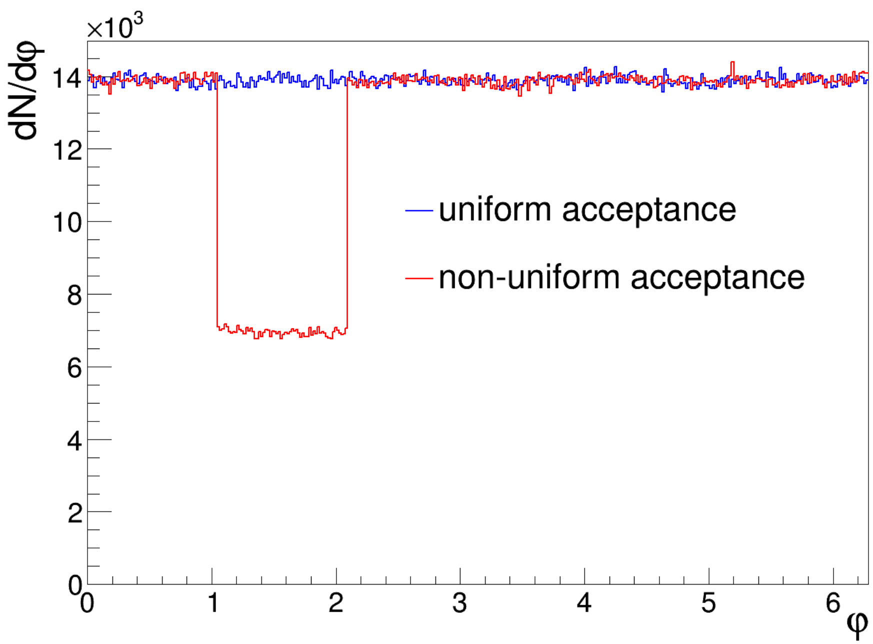
- Nonflow scaling:

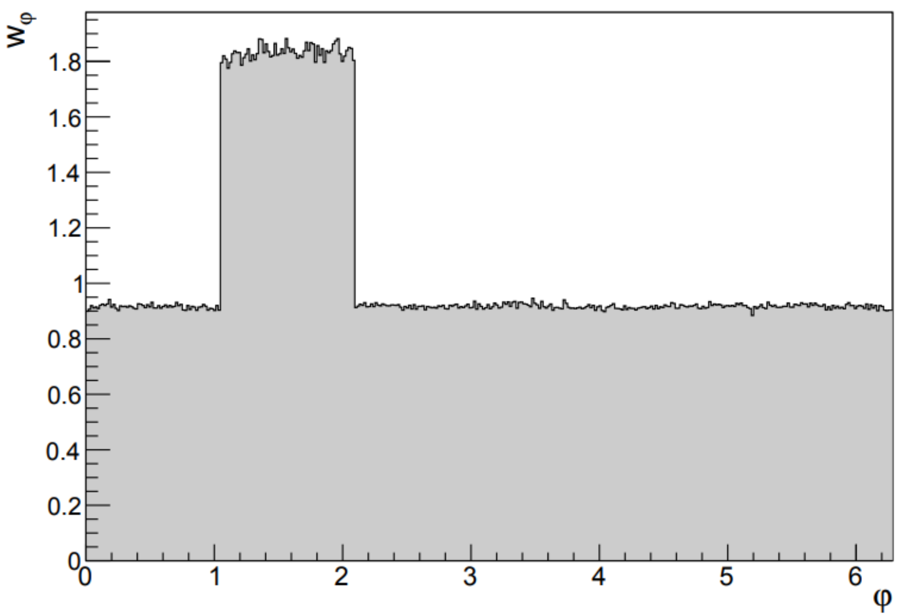
$$\delta_k \sim \frac{1}{M^{k-1}}$$

- For both reasons, multi-particle correlations is a precision technique only for: a) large multiplicities, b) large flow

Critical check for CBM #2

- Efficiency framework works as long as:
  - Particle weights can be built
  - Detector conditions are stable within each data-taking period (run), but can vary from one run to another





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Analysis code, dataset and cuts

Data format and file reader

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- **AnalysesTree** format downloaded and compiled locally:

  https://git.cbm.gsi.de/pwg-c2f/data/analysis_tree

- Compiled libraries can be loaded in ROOT 6 as:

  gSystem->Load("/usr/local/lib/libAnalysisTreeBase.so")

  gSystem->Load("/usr/local/lib/libAnalysisTreeCuts.so")

- As a starting point, I am taking file readers from Viktor:

  /lustre/cbm/users/klochkov/sand_box/analysis_tree_test/analysis_tree_simple.C

  https://git.cbm.gsi.de/pwg-c2f/data/analysis_tree/-/blob/master/examples/example.cpp

Many thanks to Viktor, Ilya and all other developers for help!

/home/abilandz/CBM/development/macrosFromViktor/analysis_tree_simple.C

Analysis source code

- At the moment, the code is in the private GitLab repository:
  - https://gitlab.com/abilandz/MultiparticleCorrelations
  - Eventually it will be ported to the central Git repository
- The analysis code currently contains:
  - Main class: FlowWithMultiparticleCorrelations.{h,cxx}
  - Macros:
    - libraries.C
    - run.C
    - mergeAndBootstrap.C
    - makeWeights.C
    - makeNonDefaultPDFs.C
- In the rest of the talk, demonstrating what the code can do with realistic and toy Monte Carlo studies

Datasets and cuts

- Realistic distributions were obtained from common Monte Carlo production with CBMROOT OCT19 release:
  - OCT19, UrQMD + PLUTO, GEANT3, Au+Au@12, PSD
    https://cbm-wiki.gsi.de/foswiki/bin/view/PWG/CommonMCproduction

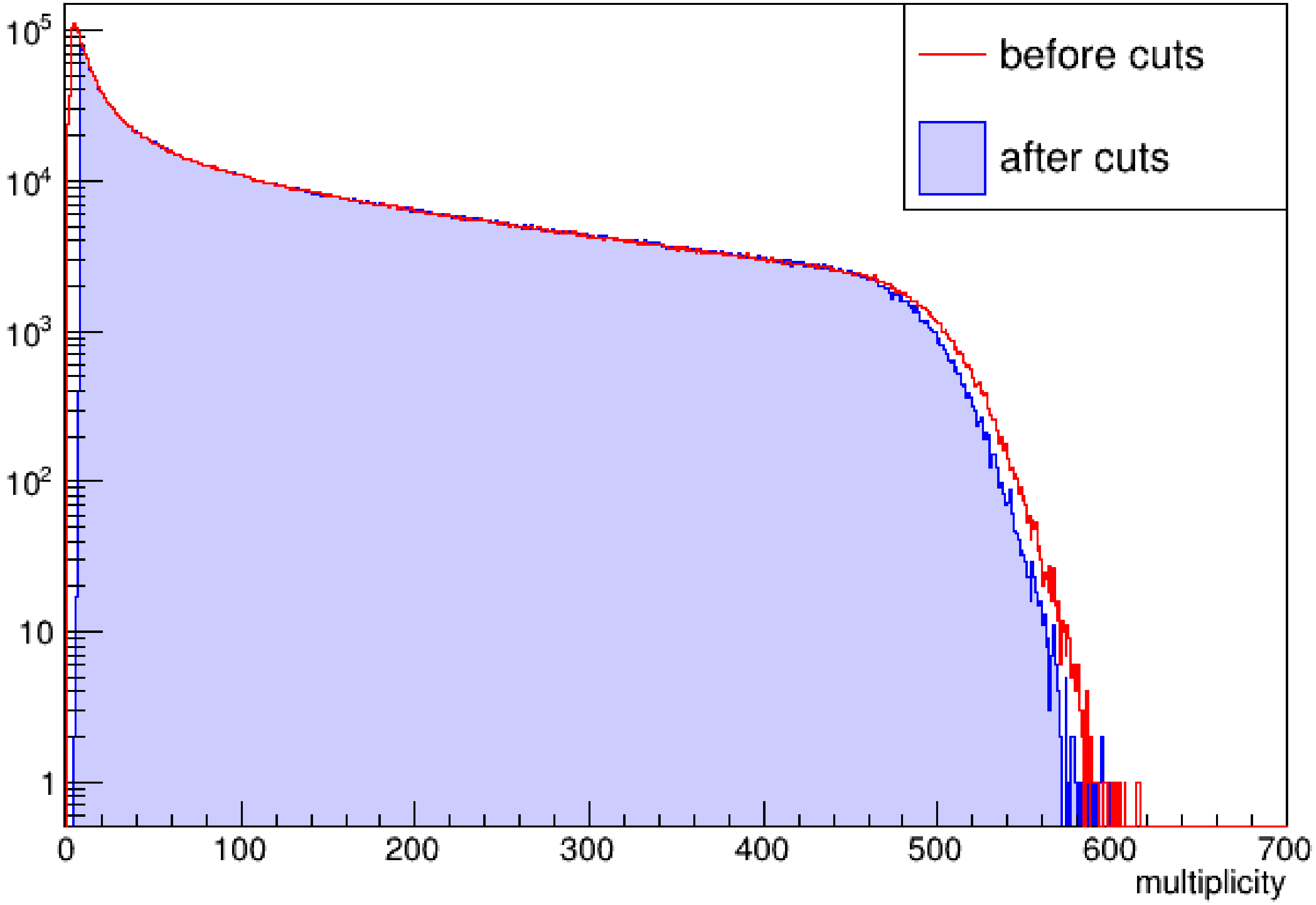
- Very basic cuts applied:
  - Rejecting all events with less than 8 particles
  - Selecting only particles with $0.0 < p_T < 5.0$ GeV
  - All vertex components in [-10,10] cm

- Final statistics: 4.06 M events

Results: Control Histograms

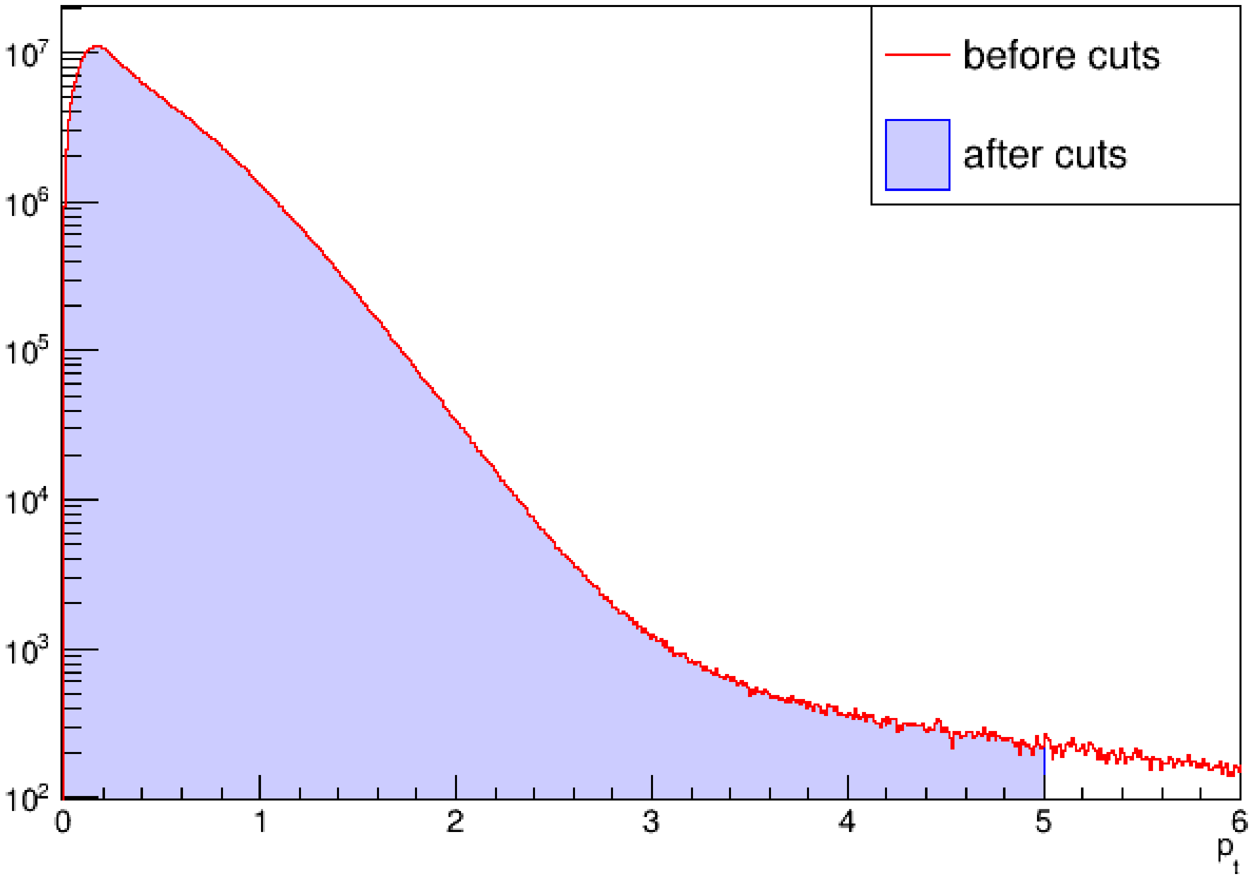
Multiplicity distribution

- Cut on large p_T removes high-multiplicity events



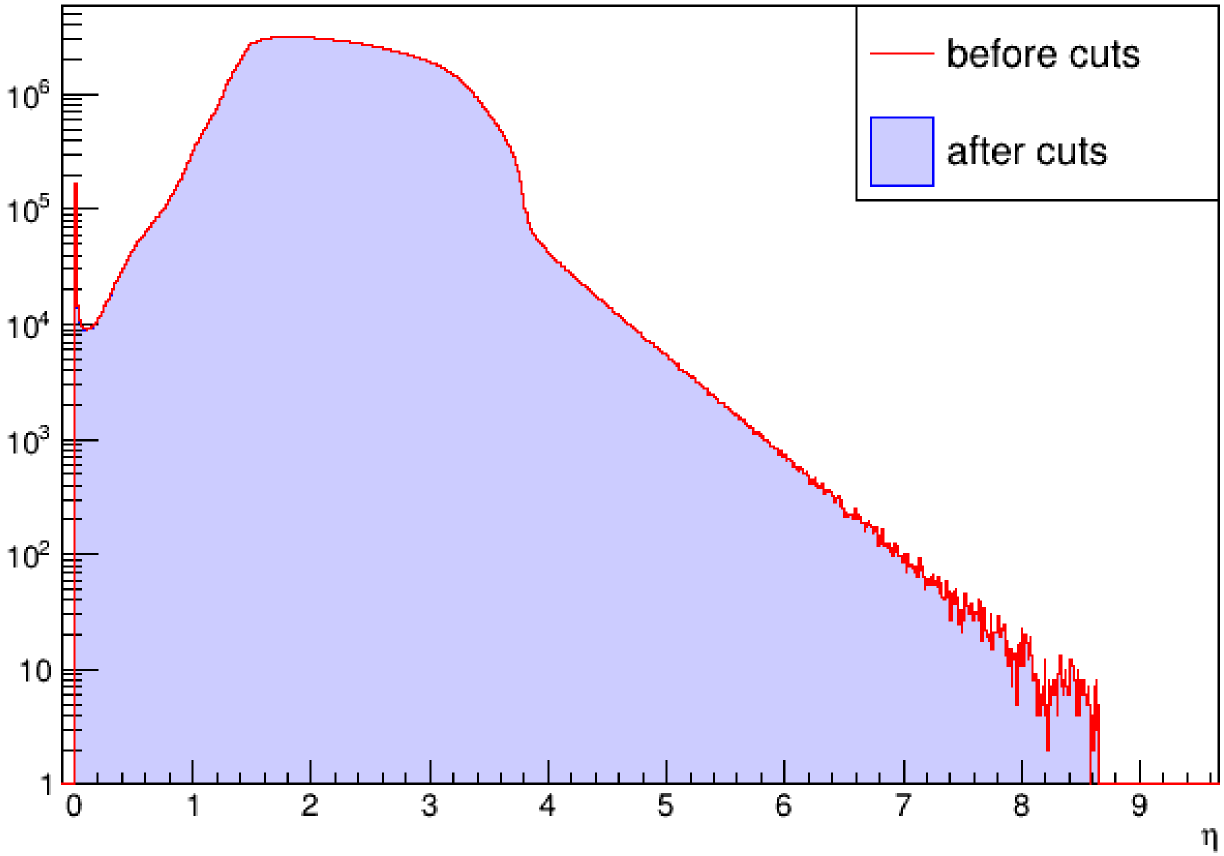
p_T distribution

- This cut needs to be optimized for flow studies:
  - all flow harmonics exhibit non-trivial p_T dependence
  - nonflow and efficiency also depend on p_T



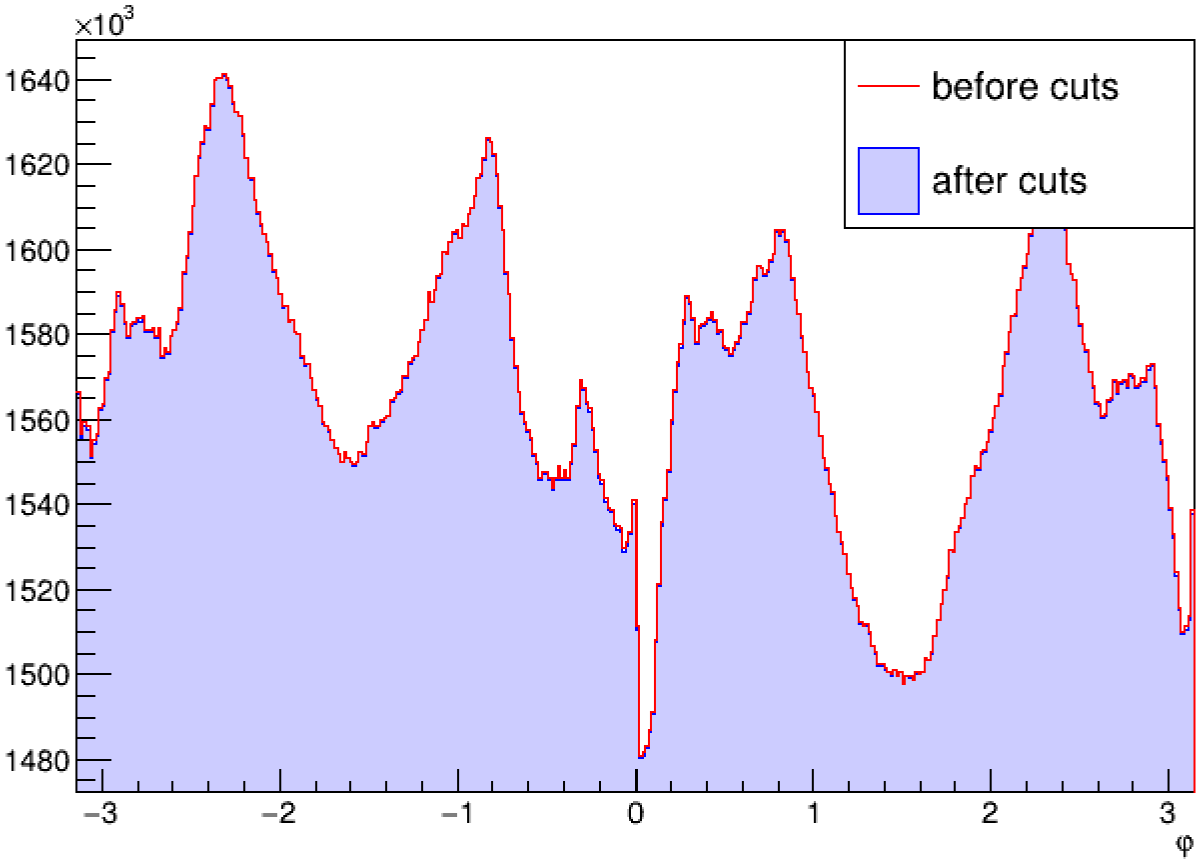
Pseudorapidity distribution

- A large asymmetry: this has a consequence on all flow analysis techniques using Δη gaps to suppress short-range nonflow correlations
  - At CBM energies, rapidity is a better variable and will be used instead



Azimuthal distribution

- Large built-in anisotropies due to non-uniform azimuthal acceptance, this will be one the most dominant systematic biases in flow analyses at CBM with correlation techniques



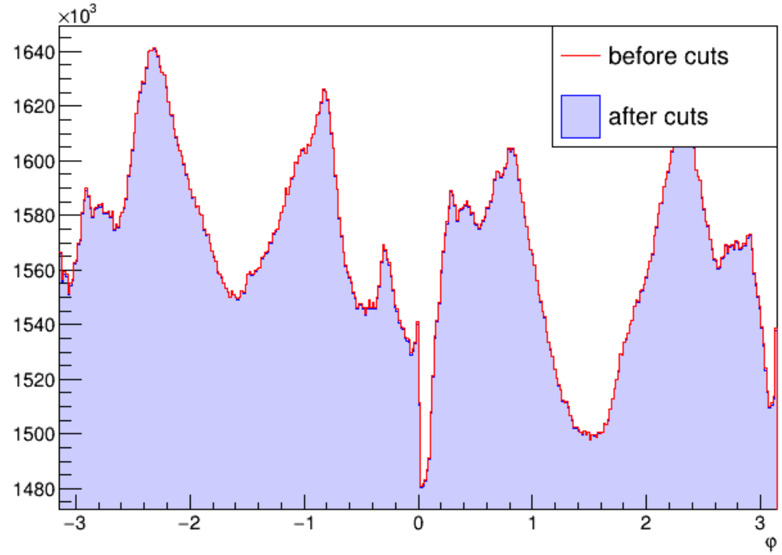
Results: Acceptance corrections

/home/abilandz/CBM/development/macrosFromViktor/analysis_tree_simple.C

Setup for Monte Carlo study

- **Question:** If there are no input values of flow, what is the spurious built-in flow which corresponds to CBM's non-uniform acceptance?
  - Can we correct for it?
- Multiplicity, particle's transverse momenta and pseudorapidity are sampled from realistic p.d.f.'s shown on previous slides
- Particle azimuthal angles are taken for the analysis with probability which corresponds to p.d.f. resembling the CBM-like azimuthal acceptance

Estimating and correcting the effect of such non-uniform acceptance on each flow harmonic v_n



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Results for spurious flow (1/6)

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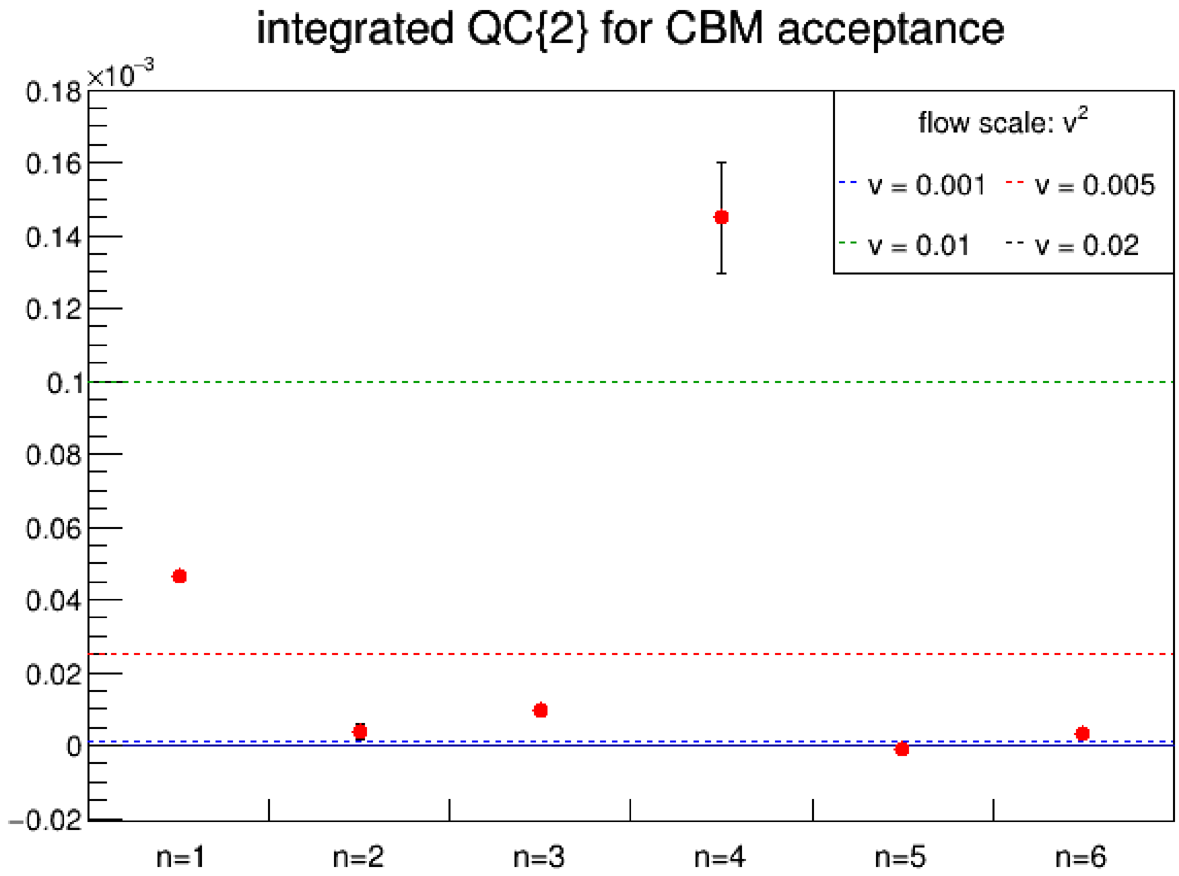
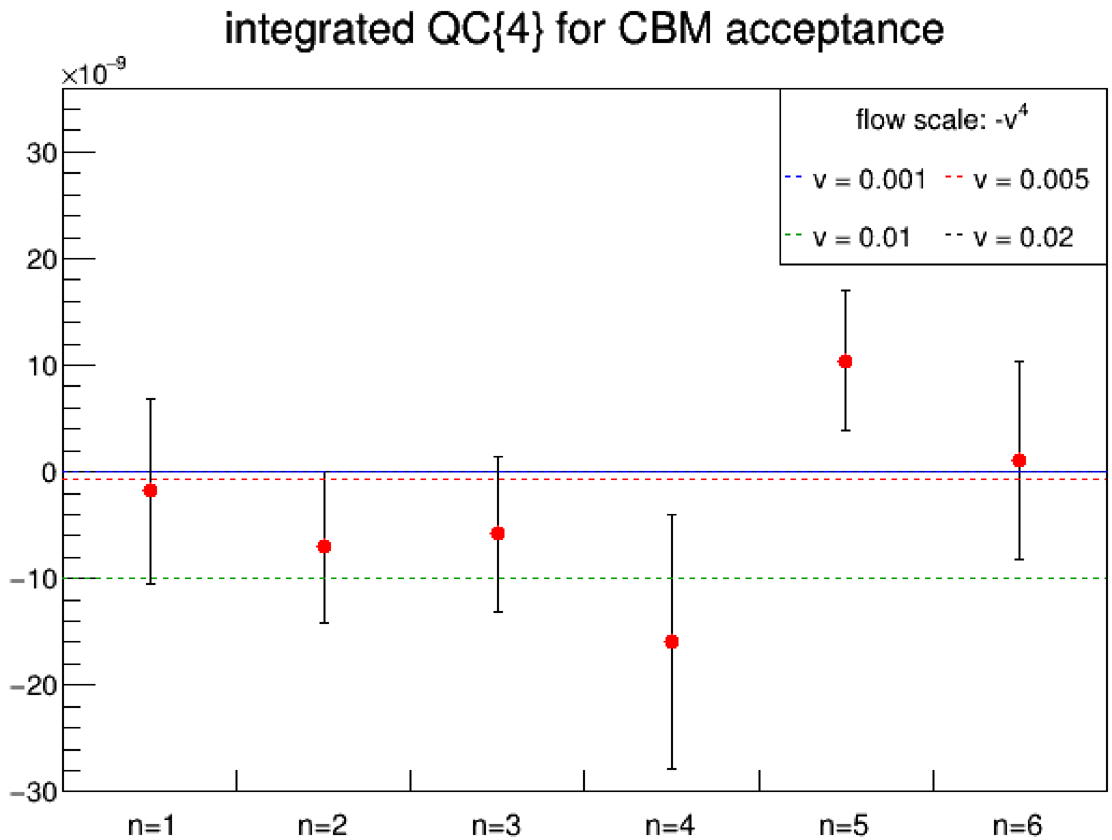
- 2-particle Q-cumulants, for CBM acceptance, for $v_1 - v_6$:



- CBM's non-uniform acceptance produces a spurious $v_1 \sim 0.5\%$ and $v_4 \sim 1\%$
- For all other flow harmonics, the effect is less than 0.5%

Results for spurious flow (2/6)

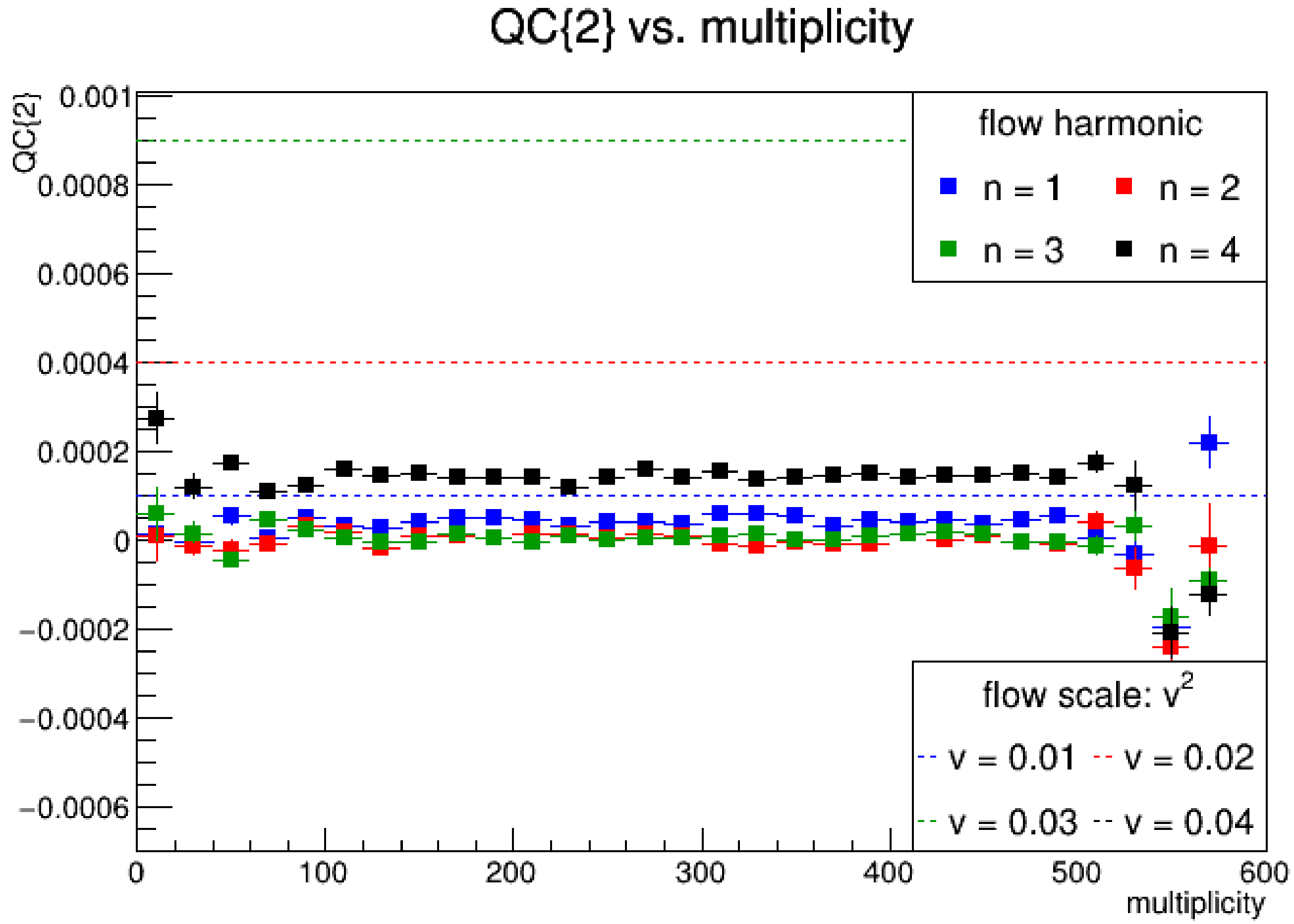
- 4-particle Q-cumulants, for CBM acceptance, for $v_1 - v_6$:



- Statistical fluctuations are larger, but results are consistent with QC{2}

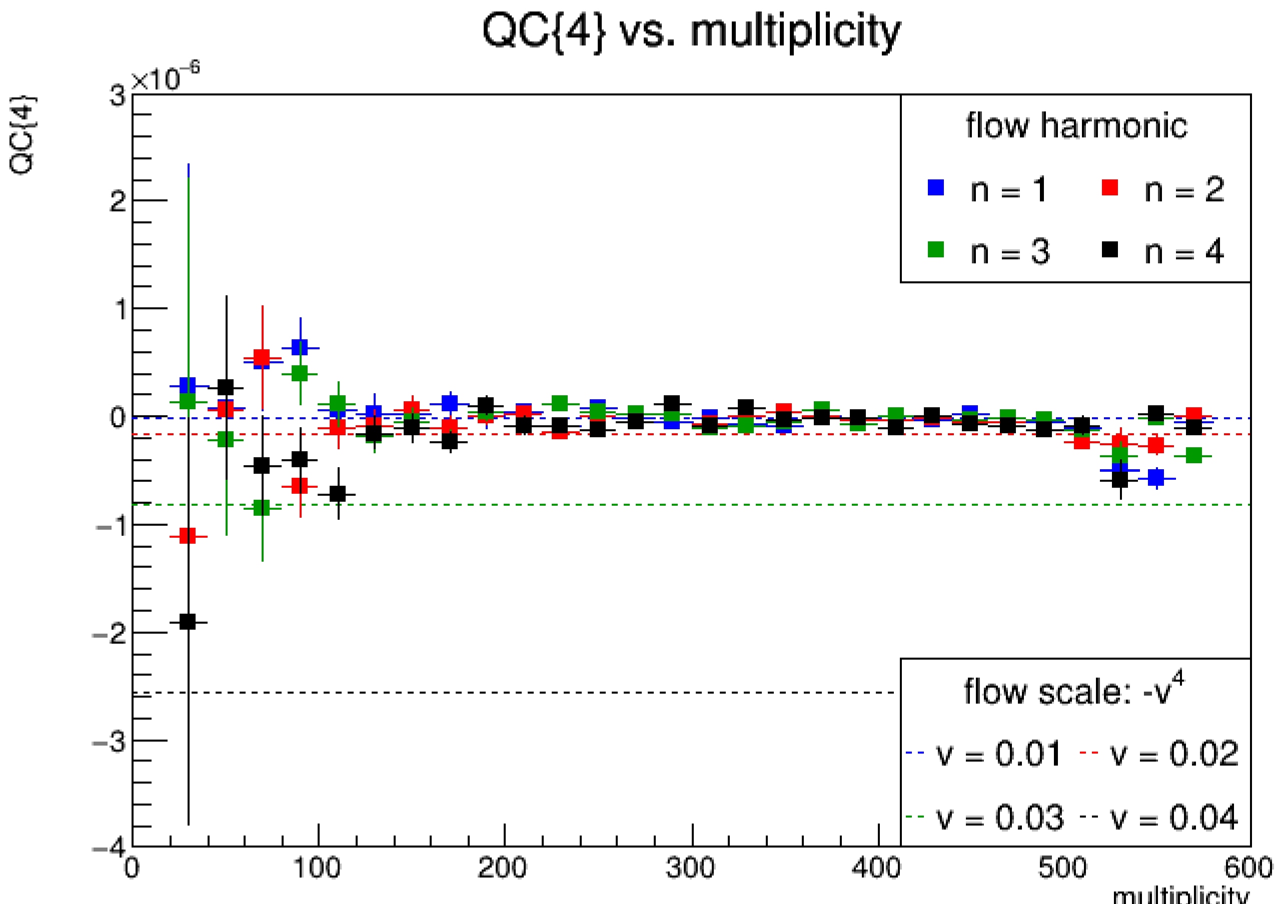
Results for spurious flow (3/6)

- Differential 2-particle Q-cumulants vs. multiplicity, for CBM acceptance, for $v_1 - v_4$:



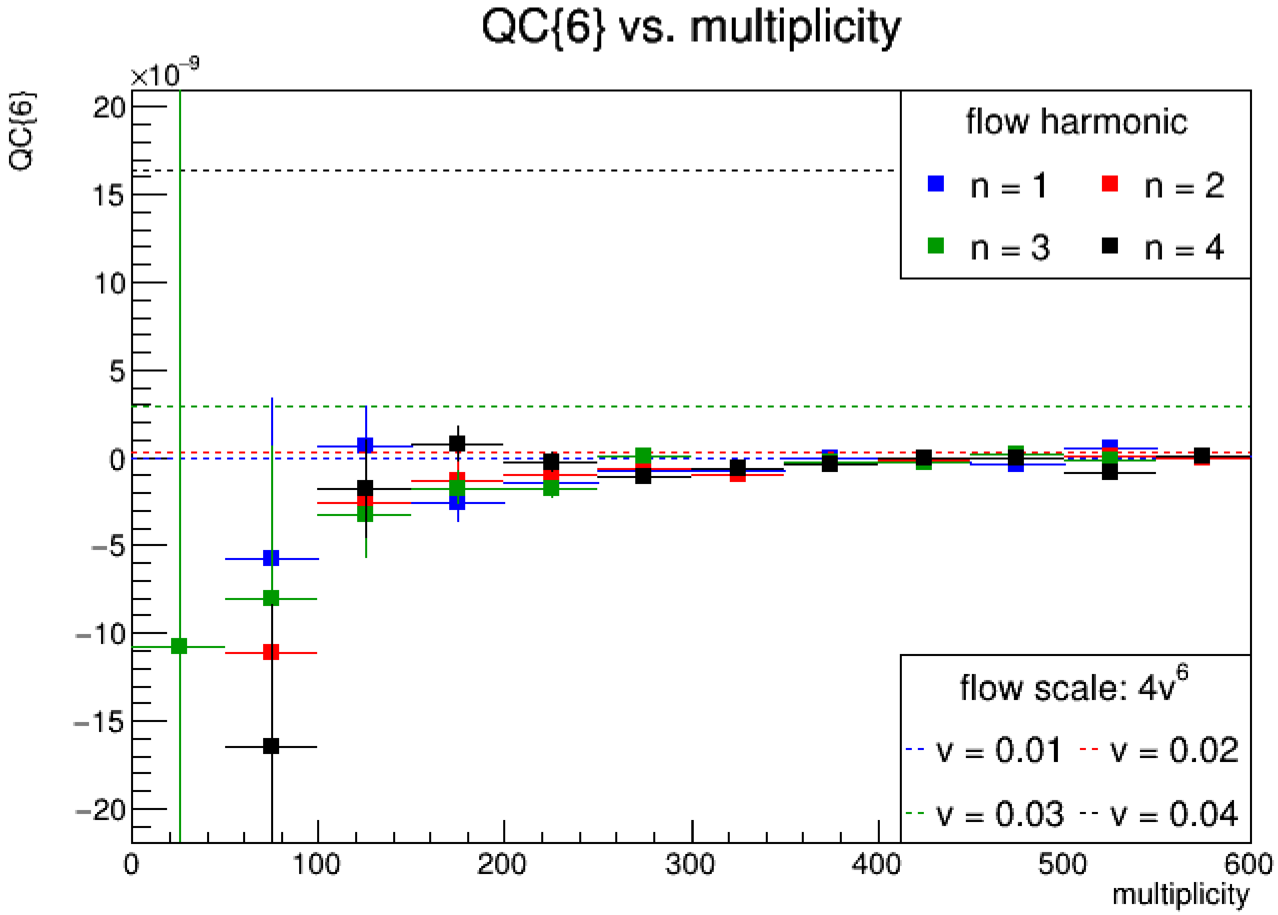
Results for spurious flow (4/6)

- Differential 4-particle Q-cumulants vs. multiplicity, for CBM acceptance, for $v_1 - v_4$:



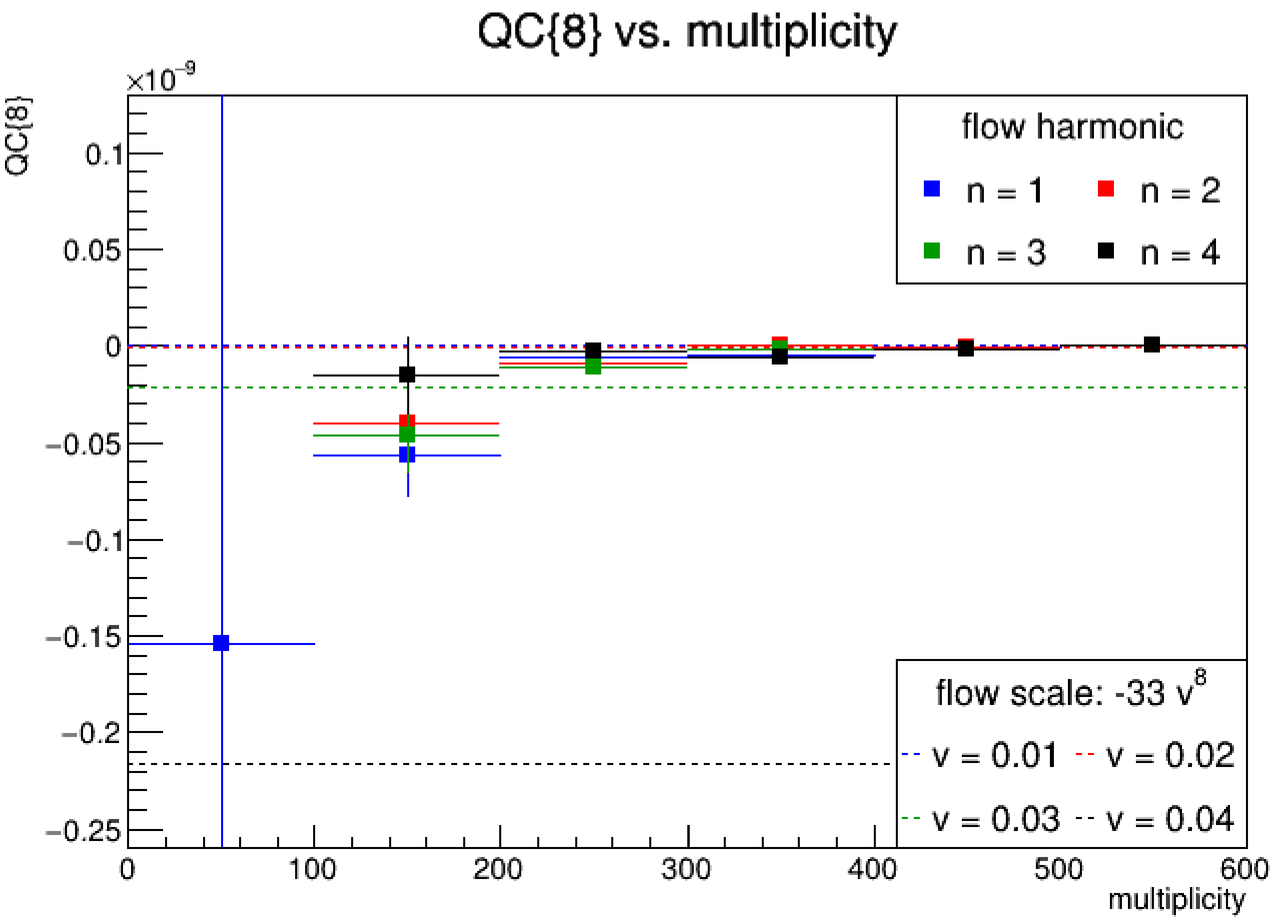
Results for spurious flow (5/6)

- Differential 6-particle Q-cumulants vs. multiplicity, for CBM acceptance, for $v_1 - v_4$:



Results for spurious flow (6/6)

- Differential 8-particle Q-cumulants vs. multiplicity, for CBM acceptance, for $v_1 - v_4$:

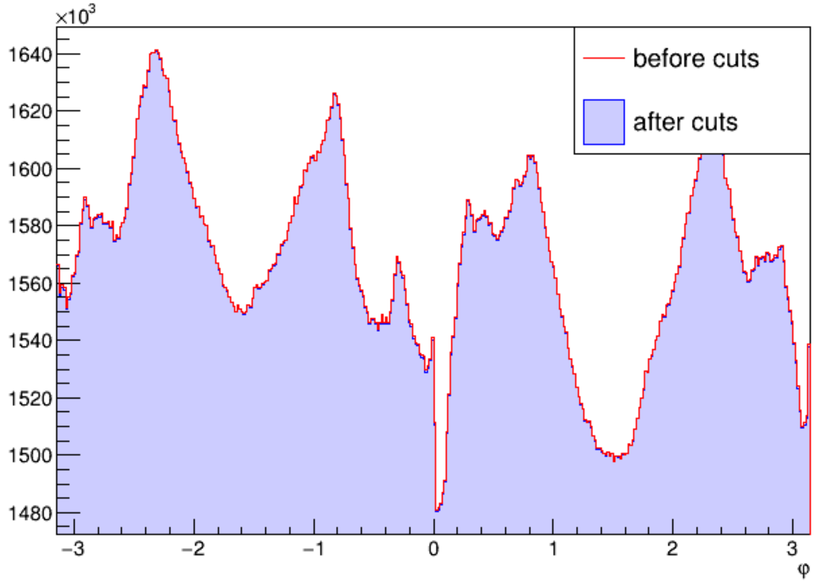


/home/abilandz/CBM/development/macrosFromViktor/analysis_tree_simple.C

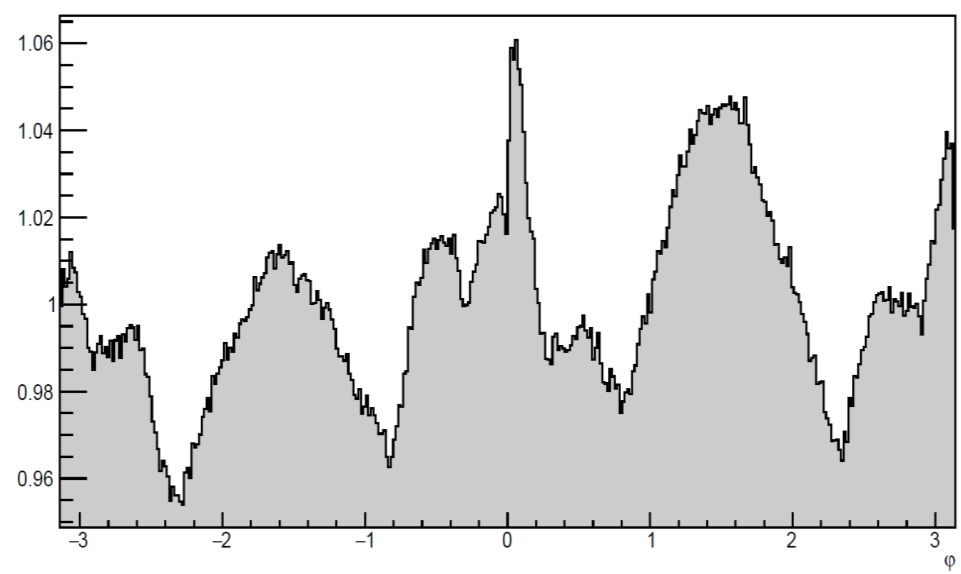
Correcting for spurious flow (1/2)

- Non-uniform azimuthal distribution needs to be inverted, to obtain φ-weights
- Rerun over the data and use φ-weights when building *Q*-vectors

$$Q_n \equiv \sum_{k=1}^{M} e^{in\varphi_k} \quad \Rightarrow \quad Q_{n,p} \equiv \sum_{k=1}^{M} w_k^p\, e^{in\varphi_k}$$



weights (inverted distribution)



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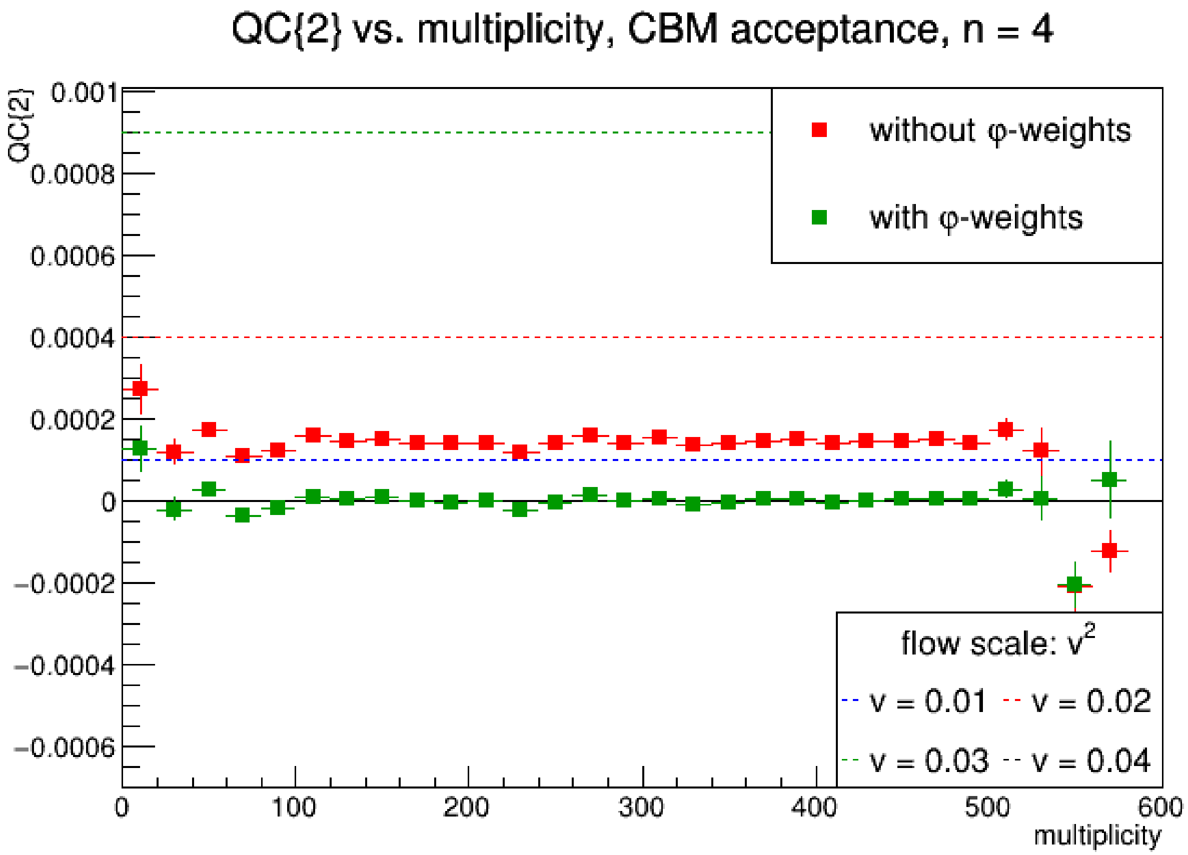
Correcting for spurious flow (2/2)

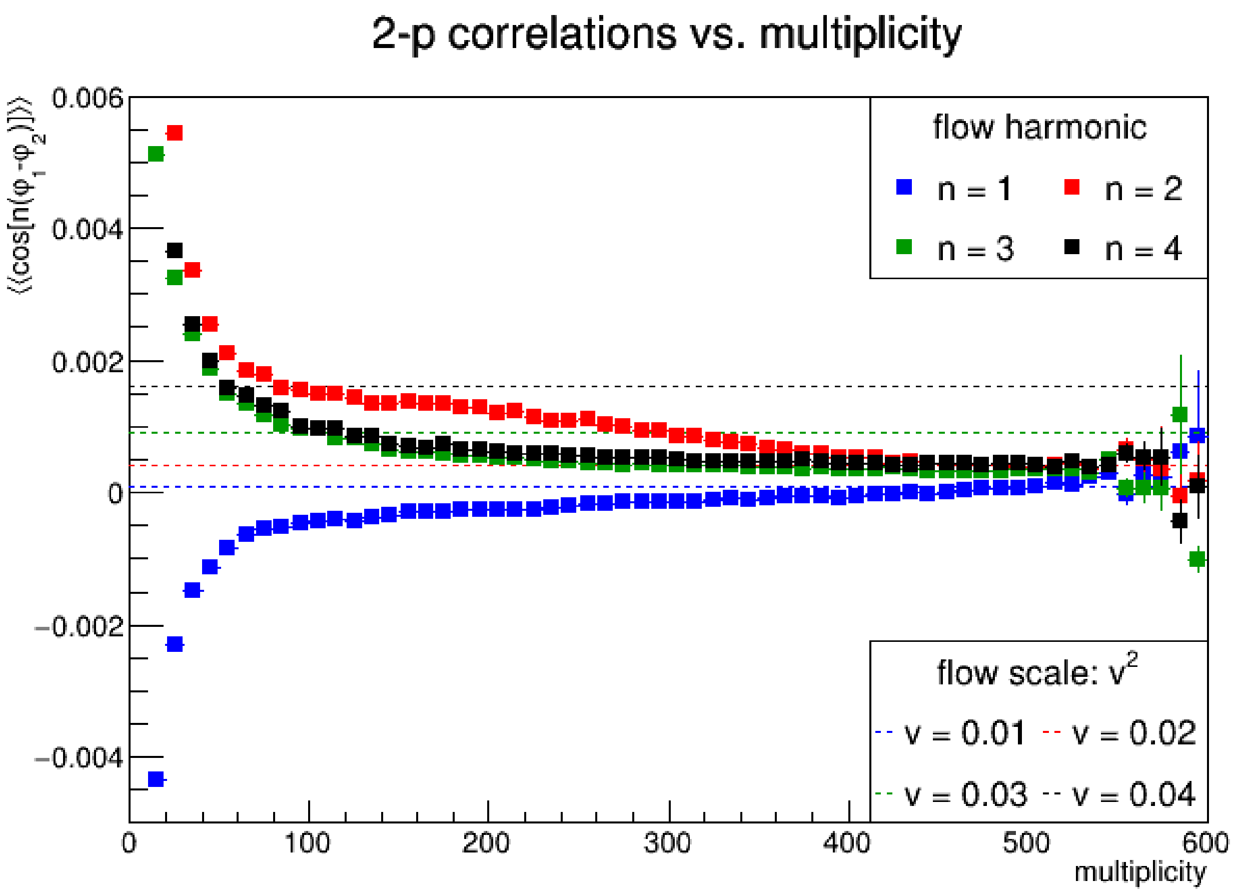
- Effects of non-uniform azimuthal acceptance in CBM are largest for v_4
  - After applying φ-weights, all results are consistent with 0!



Results: Multi-particle correlations and cumulants vs. multiplicity

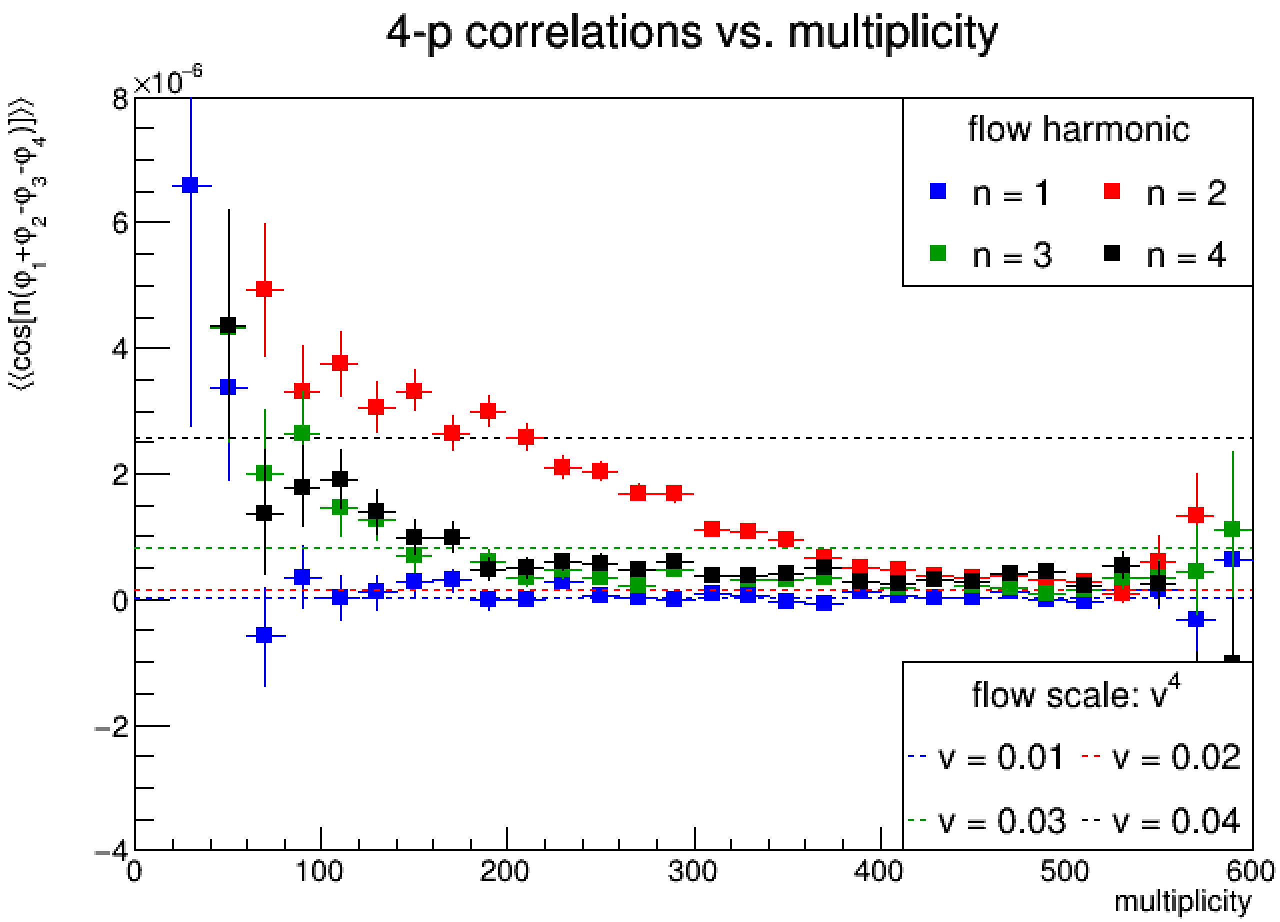
2-particle correlations vs. multiplicity

- Estimate: v_1^2(blue), v_2^2 (red), v_3^2 (green), v_4^2 (black)



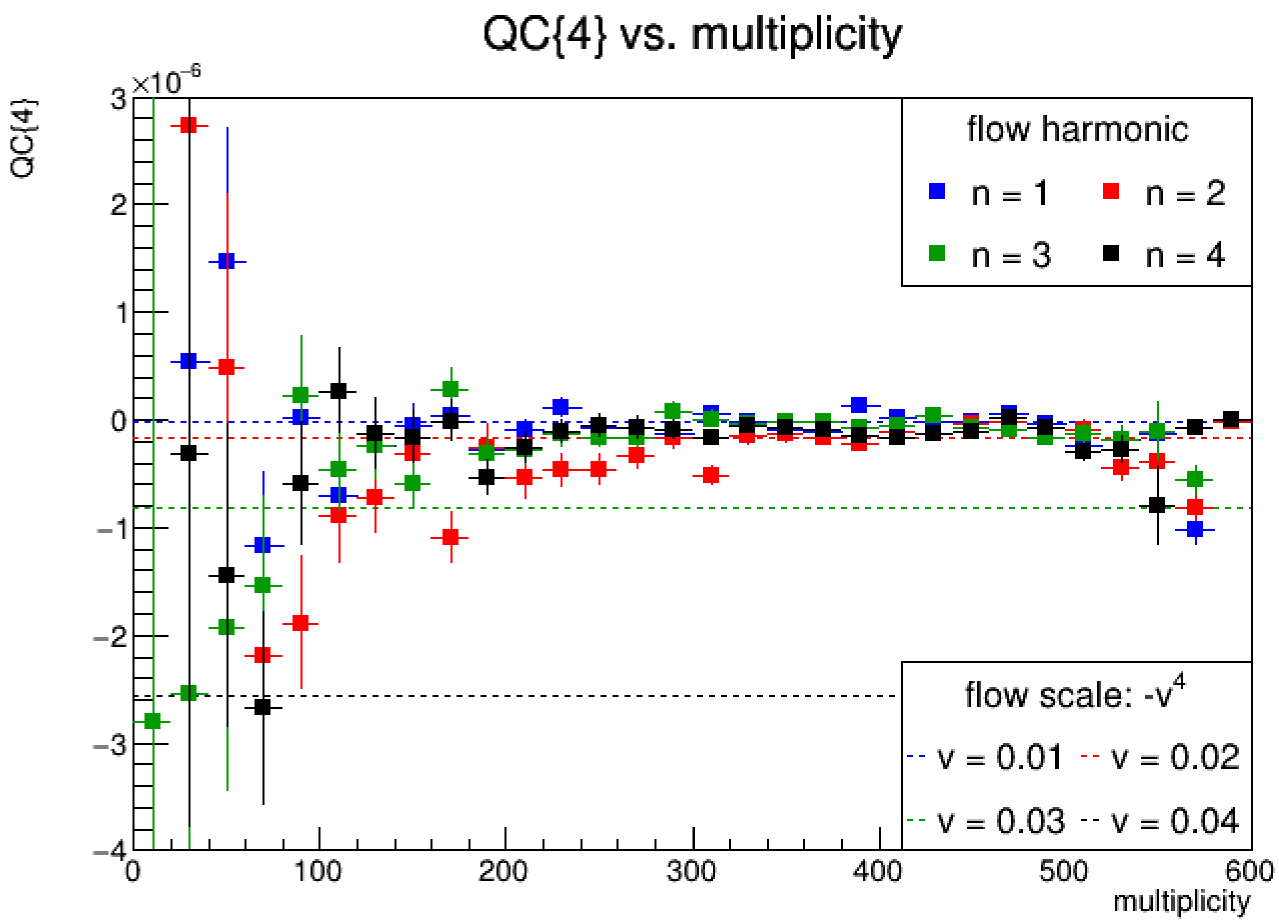
4-particle correlations vs. multiplicity

- Estimate: v_1^4 (blue), v_2^4 (red), v_3^4 (green), v_4^4 (black)



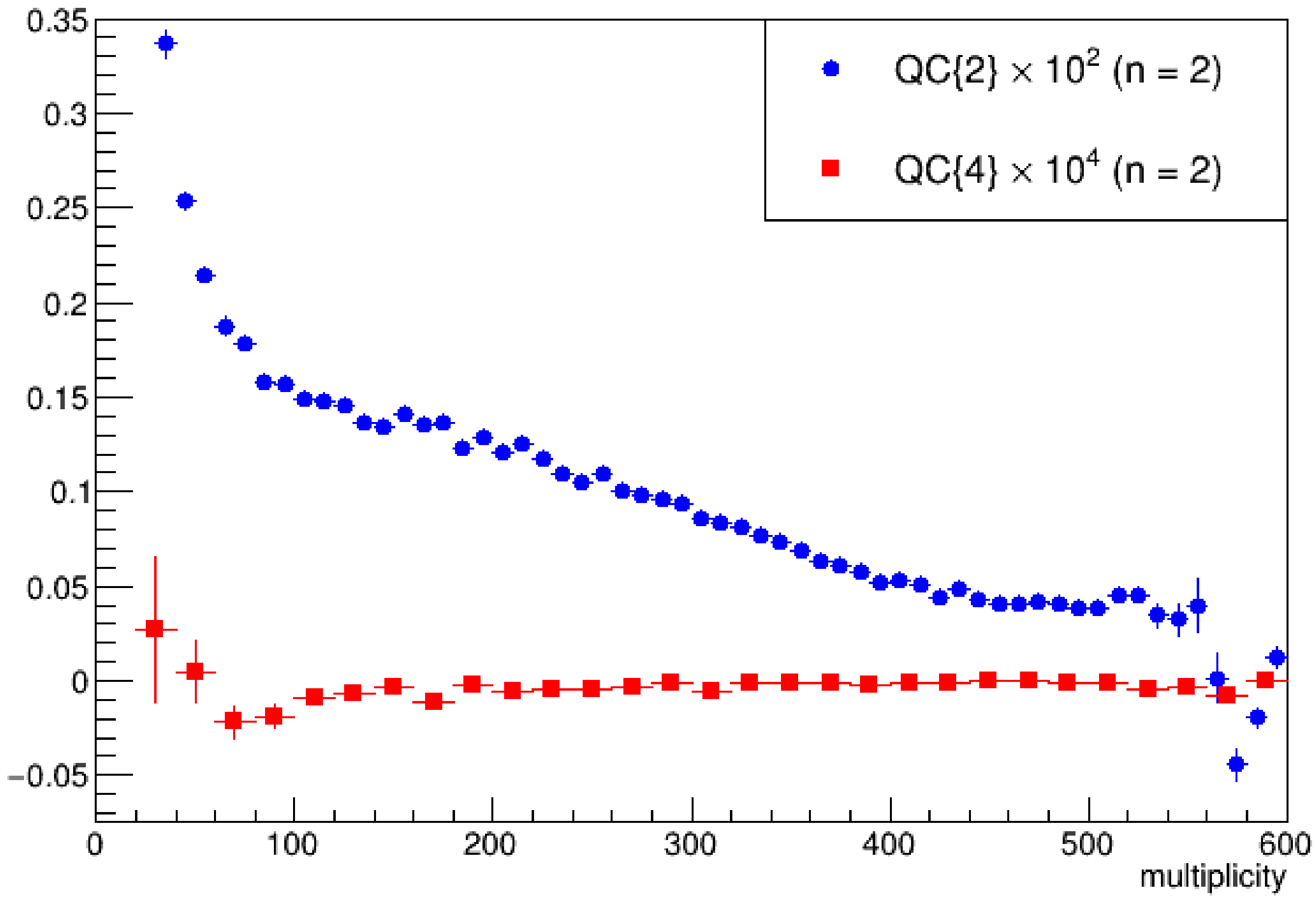
4-particle Q-cumulants vs. multiplicity

- Estimate: -v_1^4 (blue), -v_2^4 (red), -v_3^4 (green), -v_4^4 (black)



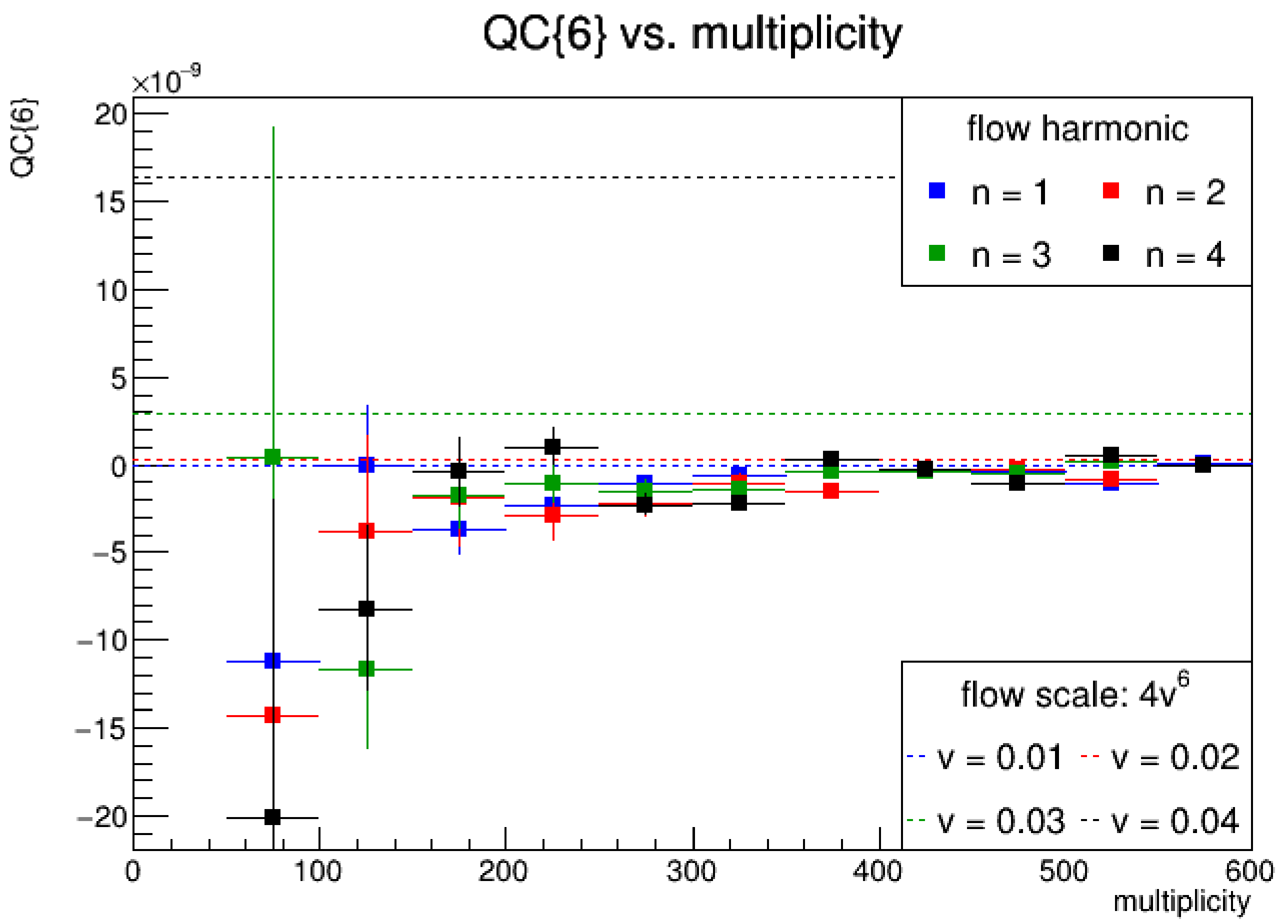
2-and 4-p cumulants at the same scale

- Estimate: $v_2^2 \times 10^2$ (blue), -$v_2^4 \times 10^4$ (red)



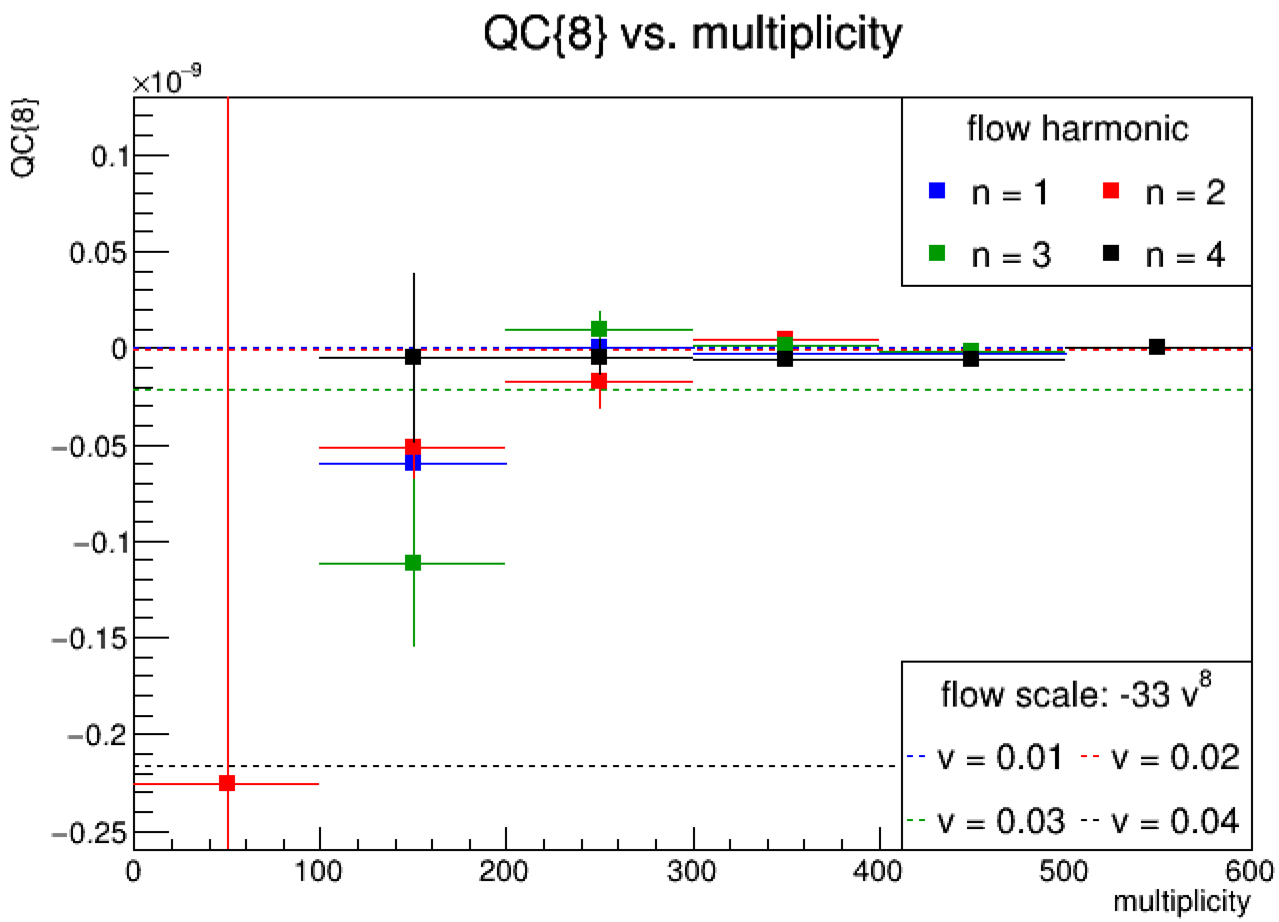
6-particle Q-cumulants vs. multiplicity

- Estimate: $4v_1^6$ (blue), $4v_2^6$ (red), $4v_3^6$ (green), $4v_4^6$ (black)



8-particle Q-cumulants vs. multiplicity

- Estimate: -33v_1^8 (blue), -33v_2^8 (red), -33v_3^8 (green), -33v_4^8 (black)



Coming next

- Monte Carlo studies with other particle weights (transverse momentum and pseudorapidity)
  - Run analysis using MC-true particles and compare with results at the reconstruction level
- PID techniques and switch from pseudorapidity to rapidity
- Moving the analysis code to the central Git repository
- Extending and optimizing event, track and PID selection criteria for CBM energies
- Adding the interface for centrality determination
- Implementing other multi-particle observables in flow analyses
  - Symmetric cumulants (SC)
  - Symmetry plane correlations
  - ...

Thanks!

Backup slides

Multiparticle correlation techniques

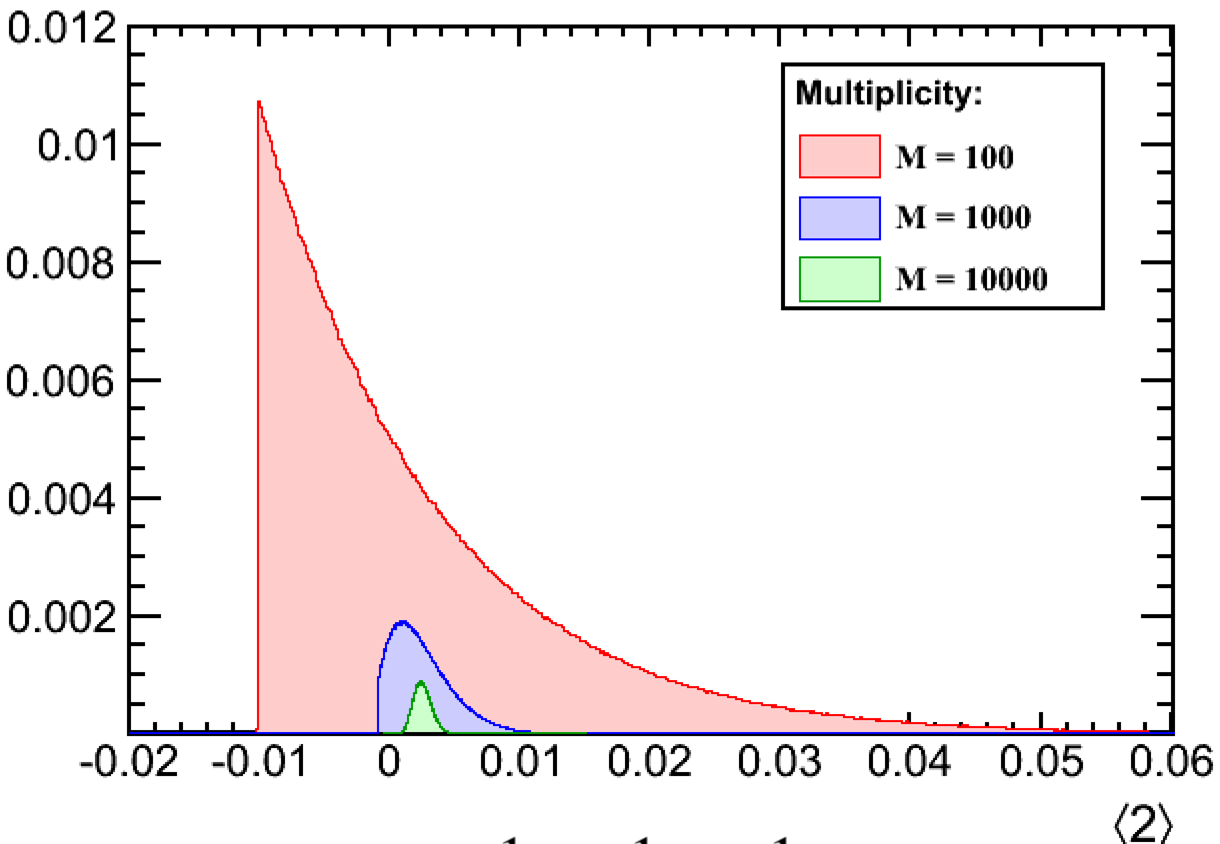
- Technical problem which plagued this field for decade: How to remove self-correlations?

$$
\begin{aligned}
\langle 2\rangle &\equiv \langle\cos(n(\varphi_1-\varphi_2))\rangle \\
&= \frac{1}{\binom{M}{2}2!}\sum_{\substack{i,j=1\\(i\neq j)}}^{M} e^{in(\varphi_i-\varphi_j)}
\end{aligned}
$$

- Formalism of generating functions developed by Ollitrault *et al* and used at RHIC is only approximate
- For data analysis at LHC we have prepared something better...

Multiparticle correlation techniques

- Monte Carlo study, fixed $v = 0.05$ as an input:



$$\sigma_v \sim \frac{1}{\sqrt{N}} \frac{1}{M^{k/2}} \frac{1}{v^{k-1}}$$

The essence of the idea

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- Estimating flow harmonics with 2-particle correlation:

event average → outer brackets; particle average → inner brackets

$$\left\langle\left\langle e^{in(\varphi_1-\varphi_2)}\right\rangle\right\rangle = \left\langle\left\langle e^{in(\varphi_1-\Psi_n-(\varphi_2-\Psi_n))}\right\rangle\right\rangle = \left\langle\left\langle e^{in(\varphi_1-\Psi_n)}\right\rangle\left\langle e^{-in(\varphi_2-\Psi_n)}\right\rangle\right\rangle = v_n^2$$

- The 'trick' works for any number of particles in the correlator
  - *k*-particle correlations estimate v_n^k
- But in the real world, there are subtleties...
  - Trivial self-correlations
  - Other sources of physical correlations ('nonflow')
  - Detector artifacts